A PRELIMINARY INVESTIGATION INTO THRESHOLD
BEHAVIOR OF PERIODIC SHOCK WAVES
IN RESONATING GAS COLUMNS

FREMONT E. REICHWEIN

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This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

PHYSICS

from the

United States Naval Postgraduate School

ABSTRACT

When an oscillating piston forces the gas in a closed
tube to vibrate at a finite amplitude near an acoustic reso-
nance frequency, shock waves are generated which travel back
and forth in the tube. The behavior of these waves in tran-
sition from the acoustic into the shock region for air at stan-
dard temperature and pressure was studied at fundamental reso-
nance frequencies up to 1000 cycles. An electrodynamic vibra-
tor, capable of variable acceleration up to 140 meters per
second per second, was used as a piston driver; it produced
acoustic pressures in the tube up to 160 db sound pressure
level.

The dependence of pressure wave shape on piston motion
was investigated; it was discovered that over a large range of
piston accelerations, nearly identical wave shapes were pro-
duced at all frequencies provided that the same piston accele-
ration was maintained.

Although the largest shocks occur when the frequency is
adjusted so that the shocks arrive at the piston just as the
piston reaches its maximum withdrawn position, shocks can be
observed at appreciable fractions of a period earlier or later.
The effect of small changes in frequency away from resonance
on this phase difference was studied, and a similarity para-
meter was determined for this variation.

Shock strength was measured as a function of both the
phase of shock occurrence at the piston and piston accelera-
tion. It was also found that acoustic resonance and shock
resonance for a given tube occur at exactly the same frequency.

TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

1. Introduction

When an oscillating piston forces the gas in a closed
tube into vibration, a number of interesting effects occur.
If one undertakes to study the motion of the gas in such a
tube according to acoustic theory, results are obtained which,
although reasonably correct in most instances, predict particle
velocities and pressures which tend to infinity at resonance.

Consider a cylindrical tube, closed at one end by a rigid
wall and at the other end by a vibrating piston. Assume that
the diameter of the tube is sufficiently small so that the
waves travel down the tube with plane wave fronts. With particle
velocity, u, as the dependent variable, the wave equation is

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

whose solution may be written

$$u(x,t) = u_+ e^{i(\omega t - kx)} + u_- e^{i(\omega t + kx)} \tag{1}$$

where $\omega = kc$, and $k = 2\pi/\lambda$, with ω the angular frequency
and λ the wavelength. The boundary conditions may be approxi-
mated by

At $x = 0$, $u(0,t) = u_o e^{i\omega t}$, so that

$$u_+ + u_- = u_o , \tag{2}$$

at $x = L$, $u(L,t) = 0$, so that

$$u_+ e^{-ikL} + u_- e^{ikL} = 0 ; \tag{3}$$

solving

$$u(x,t) = u_o \frac{\sin k(L-x)}{\sin kL} e^{i\omega t} . \tag{4}$$

If position is fixed at some particular value of x and if L is

held constant, then, as frequency is varied, both the numerator and denominator of (4) will vary. When kL is some multiple of π, the particle velocity becomes infinite, except at points where $k(L - x)$ is also a multiple of π. This leads to an expression for the length of a closed tube at resonance:

$$L\Big|_{u=\infty} = \frac{n\lambda}{2} \quad , \quad n = 1, 2, 3, \cdots$$

It may be verified by experiment that even though the amplitude of oscillation of the driving piston is infinitesimal compared to the length of the tube, large-amplitude velocity and pressure vibrations are obtained when the length of the tube is as given above. It is to be noted, however, that the approximate expression given by (4) becomes indeterminate at the ends of a resonating tube. B. Miller and L. Olsen [12] make a more nearly exact analysis based on the wave equation written with a velocity-dependent damping term. They develop an expression for the length of a resonant tube given by

$$L\Big|_{u=\infty} = \frac{n\lambda}{2} + \frac{n\lambda^5\alpha^4}{32\pi^4}$$

where α is the damping constant. In the usual experimental situation encountered in acoustics, the second term is negligible in comparison with the first, and the expression reduces to that given in the approximate analysis.

In practice, of course, although particle velocities and pressures become very large at resonance, the effects of viscosity and heat dissipation are such as to limit the amplitudes to finite values. If the piston amplitude is truly infinitesimal, the wave shape at resonance retains its sinusoidal form, and the wave may be referred to as an acoustic wave.

2

As the amplitude of piston oscillation is increased, the behavior of the gas changes. The most striking phenomenon occurs when the frequency of vibration is near one of the acoustic resonance frequencies. This is the formation of shock waves which travel back and forth in the tube. With these finite amplitude vibrations, the pressure and velocity waves make a radical departure from the sinusoidal character observed at smaller amplitudes, and it becomes obvious that the acoustic theory represented by (4) no longer applies.

Up to the present time, a complete theory describing such finite amplitude vibrations has not been formulated. Several investigations of varying thoroughness have been conducted, which have led to somewhat inconclusive results, and it appears that more experimental data are needed. Previous experiments were conducted using mechanically driven pistons which of necessity moved with constant amplitude, since without special linkages it is somewhat awkward to change the amplitude of such a driver. The experiments which are the subject of this thesis were conducted using an electrodynamic vibrator which was easily variable over a large range of amplitude and frequency, so that measurements could be made under many different conditions. This study was conducted toward a threefold purpose:

a. To show the effects of variation of amplitude on the occurrence and character of periodic shock waves.

b. To increase our knowledge of the effects of other parameters.

c. To demonstrate the feasibility of the use of an electrodynamic shaker for generating shocks for further experiment.

2. Historical Background

Investigations into the phenomena of nonlinear gaseous vibrations in a closed tube were begun in 1935 by E. Schmidt [1] and C. Mayer-Schuchard [2], who were interested in determining the nature of this type of vibration as experienced in internal combustion engines. They used a tube (length 1200 cm, radius 3.4 cm, piston stroke 3.4 cm) filled with air at normal density. A motorcycle engine was driven by an electric motor and its piston furnished excitation to the tube. Near the acoustic resonance frequencies, pressure-time histories measured at the closed end of the tube showed marked discontinuities. Between these pressure jumps the pressure records followed a curve resembling a sinusoid. Schmidt recognized the jumps as shock waves.

Using the same apparatus, E. Lettau [3] in 1939 made numerous observations of both pressure and velocity at several stations in the tube. His measurements were made by electromechanical means, and his records were obtained by photographing the trace made by a moving light beam. He found that the shape of the velocity and pressure profiles was strongly dependent on the location in the tube at which the measurement was made (see Fig. 1), and postulated an explanation involving the summation of two partial waves traveling in opposite directions.

Attempts to solve for the motion of the gas in a closed tube by mathematical means were begun somewhat later. J. Keller [4], in 1952, obtained exact solutions for the one-dimensional gas dynamic equations, assuming an inviscid, non-heat-
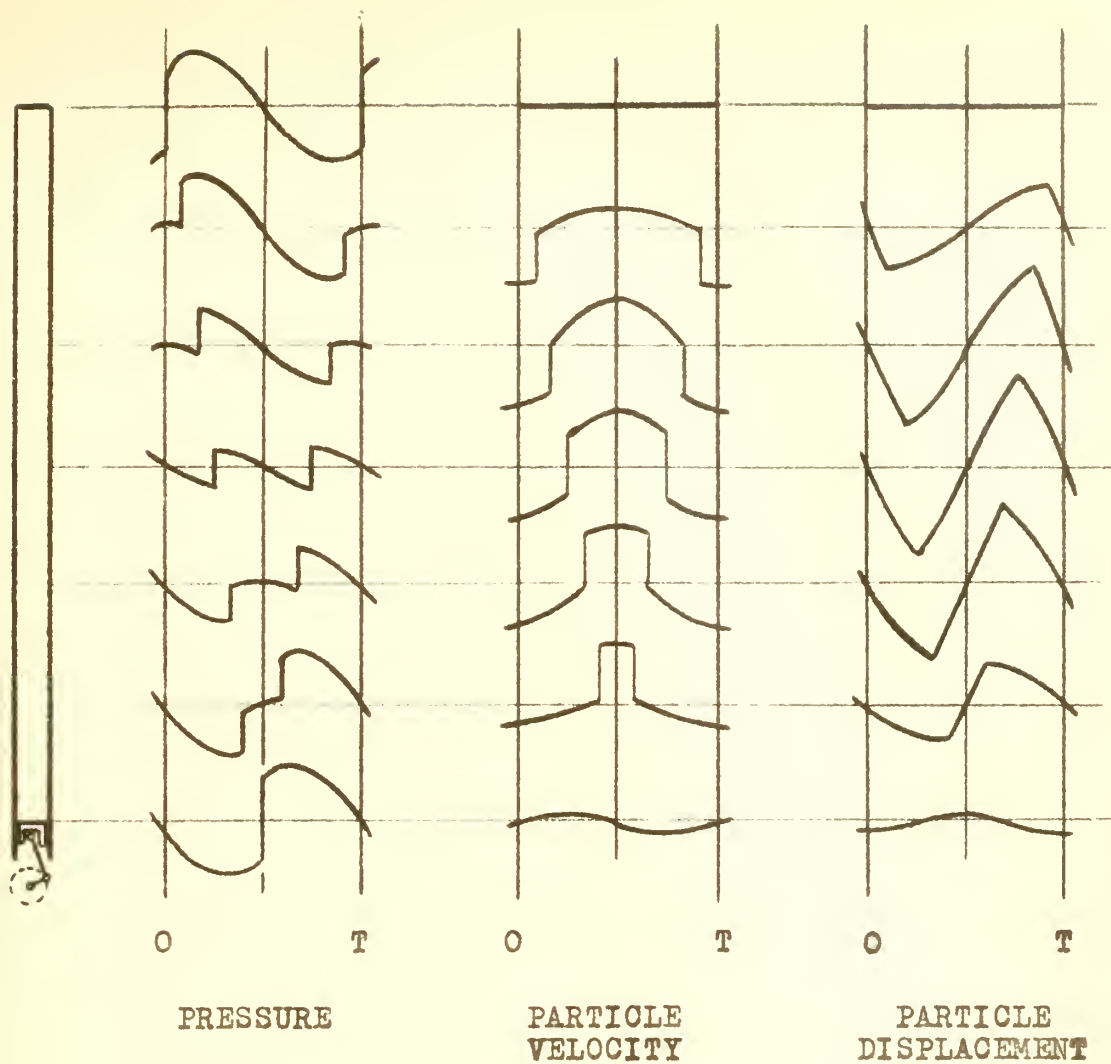
5

Figure 1.   Variation over one cycle of pressure,
             particle velocity and particle displace-
             ment at different stations in a closed
             tube resonating at its fundamental fre-
             quency. (After Lettau [3])

6

conducting medium with a particular equation of state, often referred to as a Chaplygin gas. One of the results obtained by Keller is that shocks should not occur in the tube unless the piston amplitude is in excess of $\lambda/2$, where λ is the wavelength. That experiment produces shocks at piston displacement amplitudes of considerably less than this magnitude shows that in this respect, at least, Keller's solution does not accurately describe the phenomenon.

R. Betchov [6] made a theoretical analysis of the same problem, this time assuming a tube filled with a perfect gas, and taking into account the effects of viscosity and heat conduction. He chose to make linear approximations of the motion of the gas, which he stated are valid in every portion of the tube except in the vicinity of the shocks themselves. A discontinuity between the linear approximations represents the existence of shocks. From his solution he prepared charts of the variation of velocity and density which show qualitative agreement with the results of Lettau.

R. Saenger and G. Hudson [7], [8] made a complete theoretical analysis based on the idealized physical model of a gas element shown in Fig. 2. This shows the element together with the boundary layer of gas between it and the tube wall. The element moves as a unit under the action of pressure forces on its opposite faces and frictional retarding forces at the rim. The wall acts as a heat reservoir and the use of Newton's law of cooling permits the development of one-dimensional energy balance relationships. Saenger and Hudson assumed that gas motion at resonance may be represented by the sum of a finite

7
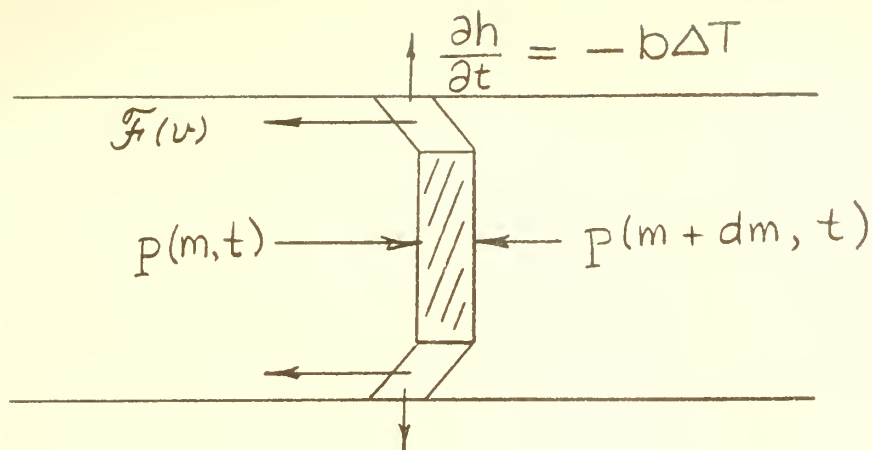
Figure 2.   Idealized physical model of gas element
             used by Saenger and Hudson for their theo-
             retical development.

continuous component and a finite discontinuous one, both of
which are periodic in time. These were obtained using the non-
linear expressions derived from the basic flow equations, the
Rankine-Hugoniot shock conditions, and the boundary conditions
on the medium. They were then used to make a theoretical deter-
mination of shock strength and mean temperature as functions
of piston amplitude, tube length and dissipation parameters.

Comparison between theory and the meager amount of experi-
mental data available at the time showed qualitative agreement.
One difficulty which faced Saenger and Hudson was the determina-
tion of the proper dissipation parameters for heat conduction
and frictional effects. They were unable to do this theoreti-
cally, and were forced to use empirical values to obtain agree-
ment between theory and experiment.

Among the conclusions obtained in their treatment is the
following: for piston amplitudes less than a critical value,
periodic shocks cannot exist. Thus for a tube of fixed length
driven at piston amplitudes below this critical value, it may

be expected that a continuous, though not necessarily acoustic, treatment is appropriate. This implies that the transition from the acoustic region to the shock region should be a sudden process, with the shock suddenly appearing fully developed in place of a wave of acoustic proportions.

3. Description of Periodic Shock Wave Phenomena

In order to observe shock wave phenomena in a closed tube, it is necessary first to match the driver frequency with the length of the tube for acoustic resonance. This length is some integral multiple of $\lambda/2$. For small values of piston displacement only the pure tone characteristic of acoustic vibrations is audible. But as the amplitude is increased, a buzzing sound is superimposed which indicates the presence of periodic shock waves. If the pressure-time history is observed on an oscilloscope, one notices the steep discontinuities occurring at the same time in each cycle. If the length of the tube is varied by a few millimeters, or the frequency by a few tenths of a cycle, the shape of the pressure curve changes drastically, and the shock wave changes in magnitude. If either frequency or length are changed by more than minimum values, the shock condition disappears and the tube returns to an acoustic mode of vibration, which gradually diminishes in strength as the tube departs further from the resonant condition.

Fig. 3 is an x - t diagram showing the motion of the shock wave as it travels in the tube, with respect to the motion of the piston. From this it can be seen that the shock wave reaches the piston at the instant that it begins the instroke portion of its cycle. In actuality, the frequency-tubelength combination may be adjusted so that the arrival of the shock at the piston is considerably out of phase with respect to this maximum outstroke position. It must be mentioned that the use of straight lines on the diagram of Fig. 3 is probably an idealization, since it is likely that they should be curved, repre-
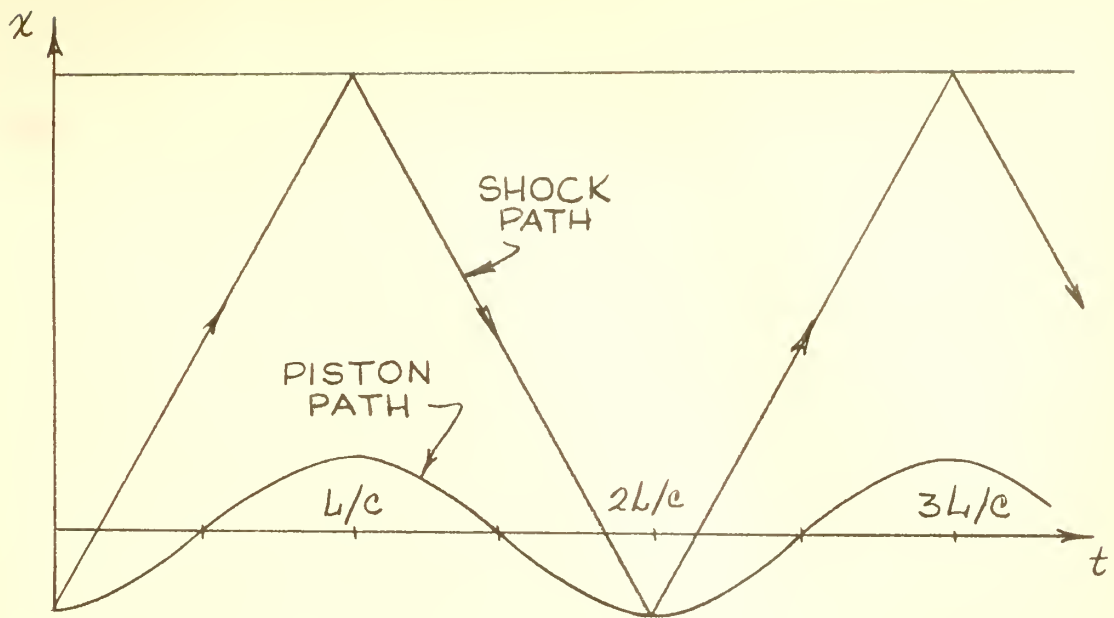
Figure 3. Space-time diagram for first resonance ($f_1 = c/2L$) showing the characteristic lines of slope $= 1/c$ along which periodic shocks travel.

senting changes in shock velocity with local conditions [6].

Particle velocity measurements can also be used to show the existence of shock waves, as can be seen from Fig. 1.

The apparatus used in the present study proved to be very well suited to the study of periodic shock waves, primarily because it allows the precise setting of parameters and the quick determination of their effects on the shock waves. The results obtained can be more easily understood in terms of the following parameters and concepts.

a. Resonance

The concept of resonance is basically an acoustical one, indicating the condition at which the tube length is precisely adjusted to be $n\lambda/2$, causing the sound waves in the tube to be of maximum strength. How the acoustic resonant condition is related to the production of maximum shock strengths had not been determined prior to the present study.

11

b. Piston Motion

In the process of making theoretical developments concerning periodic shock waves, previous writers have expressed results in terms of piston amplitude. Keller and Betchov use piston displacement amplitude in its dimensional form, while Saenger prefers to use a non-dimensionalized form which is essentially proportional to piston velocity amplitude. The results of Saenger are derived in terms of the quantity X/L, where X is piston displacement amplitude and L is the length of the tube, seeking to make theoretical predictions which are applicable to any length of tube, so long as it is resonant to the driving frequency. But

$$\lambda = 2\pi \frac{c}{\omega}$$

and for a resonant tube,

$$L = \frac{\lambda}{2} = \frac{\pi c}{\omega}$$

Hence,

$$\frac{X}{L} = \frac{\omega X}{\pi c} = \frac{V}{\pi c} \tag{5}$$

showing that the parameter X/L is actually proportional to piston velocity amplitude. Experimentally, the measurement of displacement amplitude presents some difficulty, and it is often easier to measure piston velocity and integrate to obtain displacement amplitude, which is a simple matter if the piston motion is sinusoidal.

Considerable evidence was obtained from the present study that piston acceleration is a more appropriate parameter. Acceleration can also be calculated simply from piston velocity measurements.
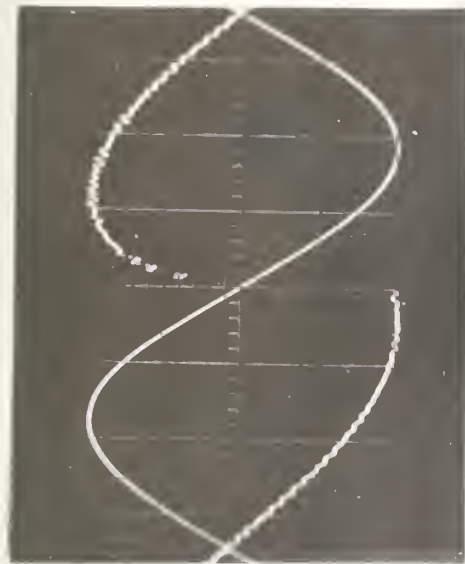
c. The parameter $\triangle$.

One of the most critical variables affecting the strength of the shocks and the shape of the waves is the phase difference between the time of the arrival of the shock at the piston and the time that the piston reaches its maximum outstroke position. This parameter, $\triangle$, is illustrated in Fig. 4. In this illustration the smooth curve is a velocity output from an electromagnetic pickup attached to the piston armature. When this curve crosses the horizontal axis, the piston is at one of its extreme positions. The phase of the extreme position can be measured with respect to the occurrence of the shock. In the present experiment, the measurement of velocity was made at the piston, and the measurement of pressure at the opposite end of the tube. One must assume that the shock takes half a period to travel the length of the tube in each direction. The parameter $\triangle$ has been assigned a negative sign if it is such that the piston reaches its maximum outstroke position before the shock appears at the piston, and a positive sign if the piston extremum occurs after the shock.

d. Tube shape and dimension.

It seems likely that tube shape and dimension would have considerable effect on the behavior of the resonating air column. The theory of Saenger and Hudson involves the effects of viscosity and heat conduction, both of which would be sensitive to changes in the tube dimension. If schlieren-optical techniques are to be used, it would be necessary to change the shape of the tube to a rectangular cross-section. In the present study, only one tube was used, and that of circular shape.

13
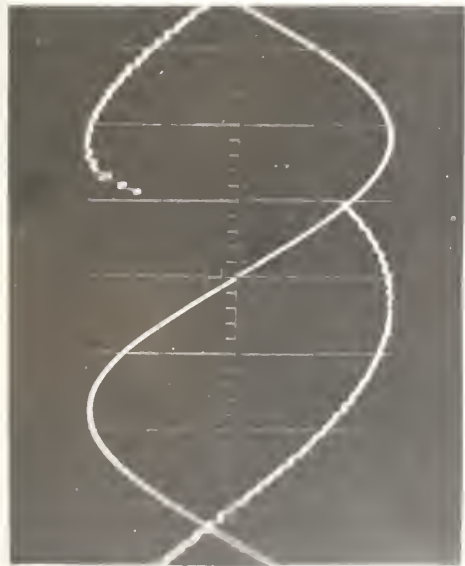
a. $\triangle = -50°$ b. $\triangle = 0°$ c. $\triangle = +45°$

Figure 4. Variation of pressure wave shape as the parameter $\triangle$ is shifted in value.

These oscillograms were taken at $f_1 = 200$ cps. The smooth curve is the output of the velocity pickup coil. The parameter $\triangle$ is the position of the zero velocity (maximum displacement) point, measured with respect to the shock position.

4. Description of Equipment

    a. Piston driver.

For the study of threshold behavior of periodic shock waves, the desirable characteristics of a driver include (1) sufficiently high amplitude for the production of shocks, (2) continuously variable frequency over a wide range, (3) easily adjusted and continuously variable piston amplitude, and (4) pure sinusoidal motion. Although it is theoretically possible to obtain all of these qualities in a mechanical driver, design difficulties arise which would ultimately result in an expensive and cumbersome apparatus which would be difficult to use.

The electrodynamic vibration shake table was decided upon as a driver closely approaching the four desirable states mentioned above. For this study the equipment used was a Calidyne Model A88 Shaker, manufactured by the Calidyne Company, Winchester, Mass. This shaker is designed with the objective of providing a light weight, yet rugged armature frame for transmitting vibratory forces to an attached load. A set of trunnions allows it to be tilted for horizontal operation. Tapped holes are provided for connecting the load to the armature of the shaker. Into one of these holes a piston was screwed which was used to excite the air column in the tube.

Included with the shake table were a Power Supply Model 68, an audio oscillator, a Power Amplifier Model 68, and a D. C. Field Supply Model 102C, all Calidyne equipment. FREQUENCY and AMPLITUDE controls on the audio oscillator, as well as a GAIN control on the power amplifier, allowed variation of frequency from 5 cps to 10 kc, and of rms acceleration from zero

15

to about 140 meters per second per second. In addition, choice of proper impedance match (using IMPEDANCE MATCH selector on Amplifier Model 68) produced sinusoidal piston motion with less than one percent harmonic distortion in the velocity signal at frequencies up to 2000 cycles. Specifications of the Model A88 shaker are included in Appendix I.

The Model A88 Shaker is equipped with a velocity pickup which consists of a coil attached to the table of the shaker so as to move in the field of a permanent magnet attached to the shaker body. The voltage produced is proportional to the relative velocity between the shaker table and body. This voltage can be continuously monitored as an indication of the absolute table velocity. The use of an a. c. voltmeter allows the shaker to be adjusted for operation at desired velocity amplitudes.

b. Microphone

An Altec 21-BR-200 microphone system with Amplifier Type 526B was used as a pressure pickup. This microphone was used because of its rugged construction and essentially flat frequency response curve. Its relatively slow rise time of about 60 microseconds placed a limitation upon the accuracy of observations made at higher frequencies. A comparison calibration of the microphone pressure sensitivity was performed using a Western Electric microphone Model WE-640-AA as a standard. The results of this calibration are contained in Appendix II.

c. Tube and Piston.

The study was performed using a 1-3/4 inch diameter aluminum tube approximately seven feet long. A simple aluminum pis-

ton of approximately the same diameter was machined, and was connected by a short threaded rod to the shaker. Initially, a metal bracket was constructed into which the tube was to be threaded; the bracket was to be bolted to the face of the shaker with the intention of holding the end of the tube rigidly in place. Because of alignment difficulties, trial runs were made with the tube loosely supported on a wooden frame, with the end floating on the vibrating piston. Using this method of support, it was found that the tube became almost self-aligning, and no other difficulties were immediately presented. Therefore the rigid support was discarded, and the method of non-rigid support was used during the remainder of the investigation.

It was also found desirable to make some changes in the piston as the experiment progressed. As originally constructed, the piston was smooth-walled and made metal-to-metal contact with the walls of the tube. Provided that the piston was kept well-lubricated (a mixture of Molykote lubricant and instrument oil was used), this construction seemed to be fairly satisfactory, and was employed for several weeks. Using this piston, it was noted that changes in alignment occasionally caused undue vibration and had some effect on the experimental results. On one occasion a combination of misalignment and insufficient lubrication led to extensive galling of the piston and tube, and it was realized that a redesigned piston was necessary.

The damaged piston was machined down a few thousandths of an inch to remove the galling, and incidently reducing its diameter so that metal-to-metal contact would be avoided. A groove was cut into which an O-ring could be fitted, and the piston was

17

reinstalled on the shaker. The reconstructed piston proved to be more satisfactory than the old one, since it was even easier to align, and was more nearly gas-tight than before. The O-ring was well lubricated with vacuum grease to keep wear to a minimum.

The end of the tube opposite from the piston was terminated with a movable aluminum plug which was bored so that the microphone could be mounted flush with its face. Two O-rings provided an effective seal, and a long rod threaded into the rear of the plug allowed the length of the tube to be varied.

d. Oscilloscope

A Tektronix Model 545 oscilloscope was used as a recording device because it possesses certain highly desirable characteristics. Since this study involved the measurement of steeply-rising pressure gradients, a fast oscilloscope rise time was essential; the Tektronix scope has a 0.01 microsecond rise time. The use of the plug-in preamplifier unit type CA, which provides a dual trace, allowed simultaneous observation of the pressure signal from the microphone and the shaker velocity signal. The delay feature of the Tektronix together with its sweep magnifier allowed measurement of phase differences accurate to about two degrees.

e. Frequency counter

Because of the extreme sensitivity of the experiment to frequency, it was found necessary to measure frequency to the nearest tenth of a cycle per second. For this purpose, a Hewlett-Packard Frequency Counter Model 521C was used. Using a ten-second count, the required precision in frequency was obtained.
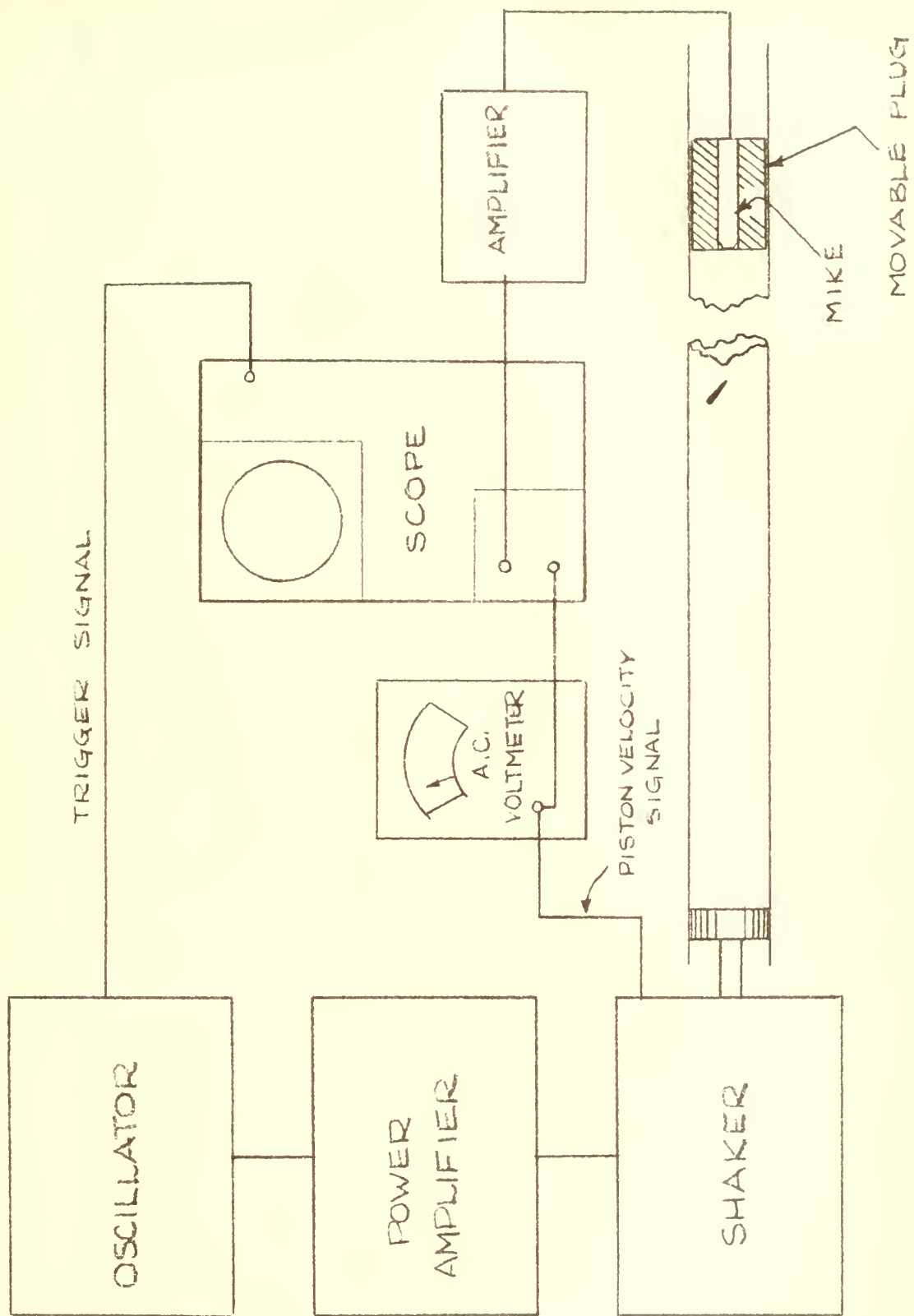
Figure 5. Arrangement of Equipment.

19

5. Experiment and Results

a. Preliminary Observations

Once the equipment was set up, a series of preliminary observations over a wide range of amplitude and frequency were made. The following are among the facts immediately noticed:

The transition from the acoustic domain to the shock domain as amplitude is increased is a gradual one. Once the frequency-tubelength combination is properly adjusted for the occurrence of shocks, increasing piston motion from zero amplitude produces a sinusoidal pressure output which gradually distorts. The first indication of an incipient shock wave is the occurrence of a discontinuity in the first derivative of the pressure-time curve. Further increase in amplitude increases the bend until it becomes vertical. Still further increase brings about a break in the curve which is the characteristic appearance of the wave containing the shock.

Although the shape of the pressure-time wave changes drastically during the transition from the infinitesimal- to the finite-amplitude region, wave shapes of almost identical appearance can be produced at each frequency for a given Δ-value by selecting the proper piston amplitude. In addition, when the shaker is oscillating at its maximum available amplitude, the pressure wave shapes obtained at different resonant frequencies are approximately the same. Using the Altec microphone, the above-mentioned effect is frequency-limited, because as frequency is increased, the rise time of the instrument assumes a more and more significant fraction of the period of vibration, masking whatever similarity may exist.

The value of $\triangle$ is a very sensitive function of both f and L. For a system adjusted so that $\triangle = 0$, at a frequency of, say, 200 cycles, a change of two-tenths of a cycle will cause $\triangle$ to change by about ten degrees, and a change of three cycles per second will drive $\triangle$ beyond 90°, whereupon the shock disappears. Small changes in the length of the tube have a corresponding effect on $\triangle$.

Shocks are found to exist in the tube under the most favorable conditions for their production (i. e., large amplitude driver motion) through a $\triangle$-range of from about -110° to +90°. As $\triangle$ passes outside of this range, the shock diminishes in size and finally disappears, although the vibration does not immediately become sinusoidal.

b. Analysis of Wave Shape.

The fact that similar wave shapes can be produced at each frequency by selecting the proper amplitude led to the supposition that there may exist a parameter, independent of frequency, by which wave shape may be specified. In order to pursue this supposition, a harmonic analysis of the wave shape was conducted for several values of amplitude and frequency. This analysis was made using a Hewlett-Packard Wave Analyzer Model 302A. Several sets of data were taken for frequencies of from 100 to 500 cps and for the second through the twelfth harmonic, as functions of rms piston velocity which was read in millivolts output of the velocity pickup. These data were then plotted against piston displacement, velocity and acceleration. It was soon noted that, although the curves of percent harmonic versus displacement and versus velocity bore little similarity

21

to each other, the curves of percent harmonic versus piston acceleration were strikingly similar, and in some cases identical, for all of the frequencies tested. It could then be postulated that the similarity parameter connecting the shocks is piston acceleration.

In an effort to substantiate this postulate, a series of twenty-seven oscilloscope photographs were taken of the pressure-time wave as seen at all combinations of three different frequencies, three different values of Δ, and three different acceleration amplitudes. In order to make a visual comparison, the oscilloscope vertical sensitivity and sweep time were adjusted so that the wave was exactly the same height and the same length in all cases.

Extraction of the data from the oscilloscope photographs was accomplished using a projection technique. Each photo was placed in an opaque projector and the curve projected onto a large sheet of graph paper. Using arbitrary numerical scales along the vertical and horizontal axes, the ordinates of the pressure-time curves were read and recorded for a number of equispaced points along the abscissa. Initially, seventy-two points were read off each curve, but this was later reduced to fifty.

The harmonic analysis itself was done using the CDC Model 1604 digital computer installed at the U. S. Naval Postgraduate School. A program was written which would accept input data in the form of the data points from the curves, perform the Fourier analysis, and then divide each amplitude by the amplitude of the fundamental. In this way, the results were in

effect normalized to a common reference, in the same way as for the work done with the electronic wave analyzer.

The data obtained in this way were then plotted, with percent of the fundamental amplitude as ordinate, and root mean square acceleration, $\mathcal{A}$ as abscissa. If acceleration is indeed, as postulated, a similarity parameter, then for a given value of $\triangle$, the points taken for common $\mathcal{A}$ should be the same regardless of frequency. The results obtained in this portion of the experiment are plotted in Figs. (6), (7), and (8), and show that agreement was obtained to within at least two percent in every case, and thus tend to confirm the basic postulate. It is believed that departures from the anticipated agreement are due in part to small fluctuations in the value of $\triangle$ for the points of different frequency and to small changes in microphone sensitivity to the different harmonic frequencies. Even though perfect correlation was not obtained, it must be realized that if the harmonic amplitudes were plotted with respect to a parameter other than acceleration (such as velocity amplitude or displacement amplitude), the curves would be so dissimilar that they could not be plotted on the same piece of paper.

In order to demonstrate further the meaning and effect of acceleration as a similarity parameter, two sets of photographs taken at different piston frequencies are included herein for comparison. The first (Fig. 9) shows a pair of oscillograms of the pressure-time wave taken at the same value of piston velocity amplitude, and the second (Fig. 10) shows a similar pair taken at the same value of piston acceleration amplitude.

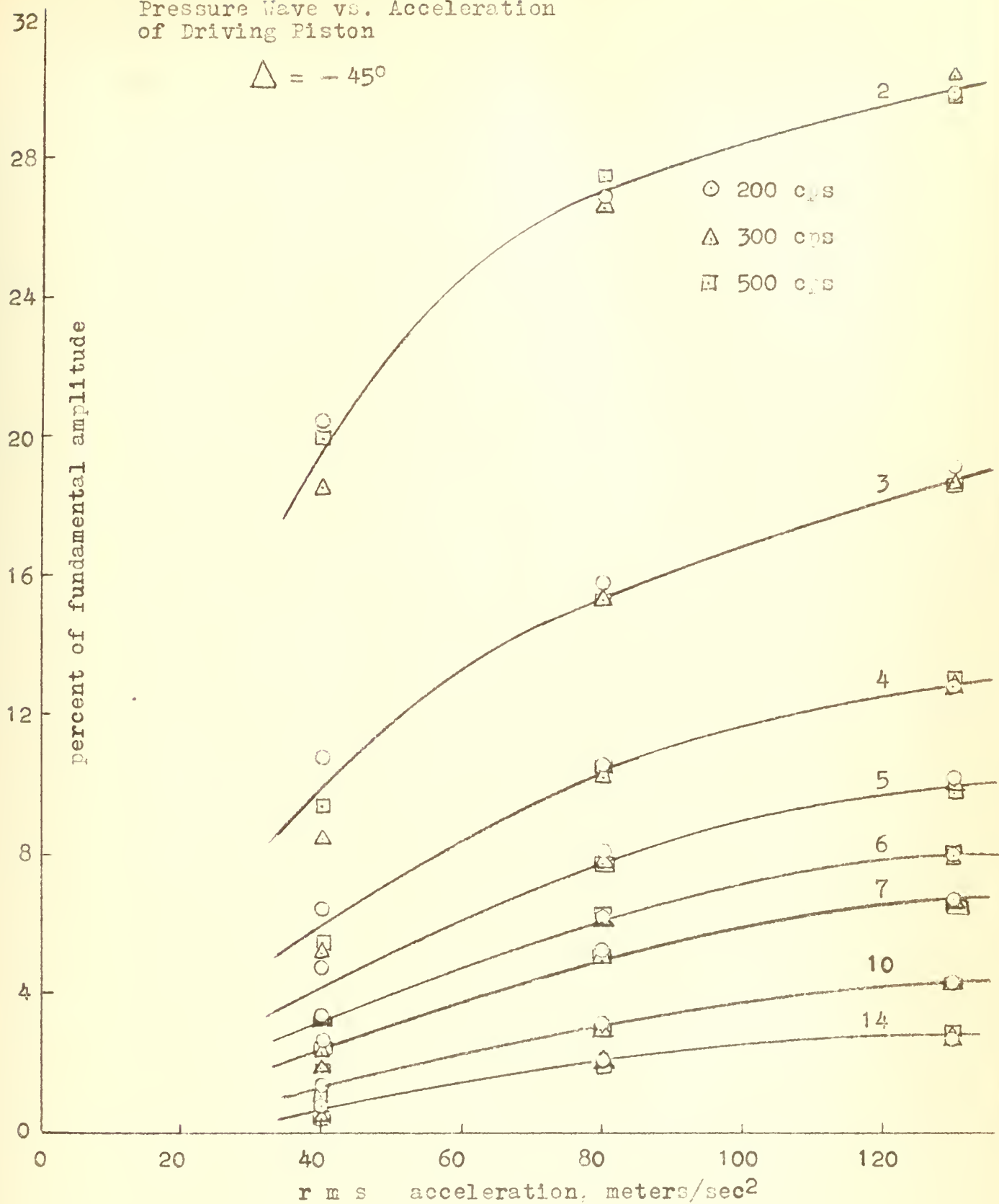Figure 6. Harmonic Content of Pressure Wave vs. Acceleration of Driving Piston
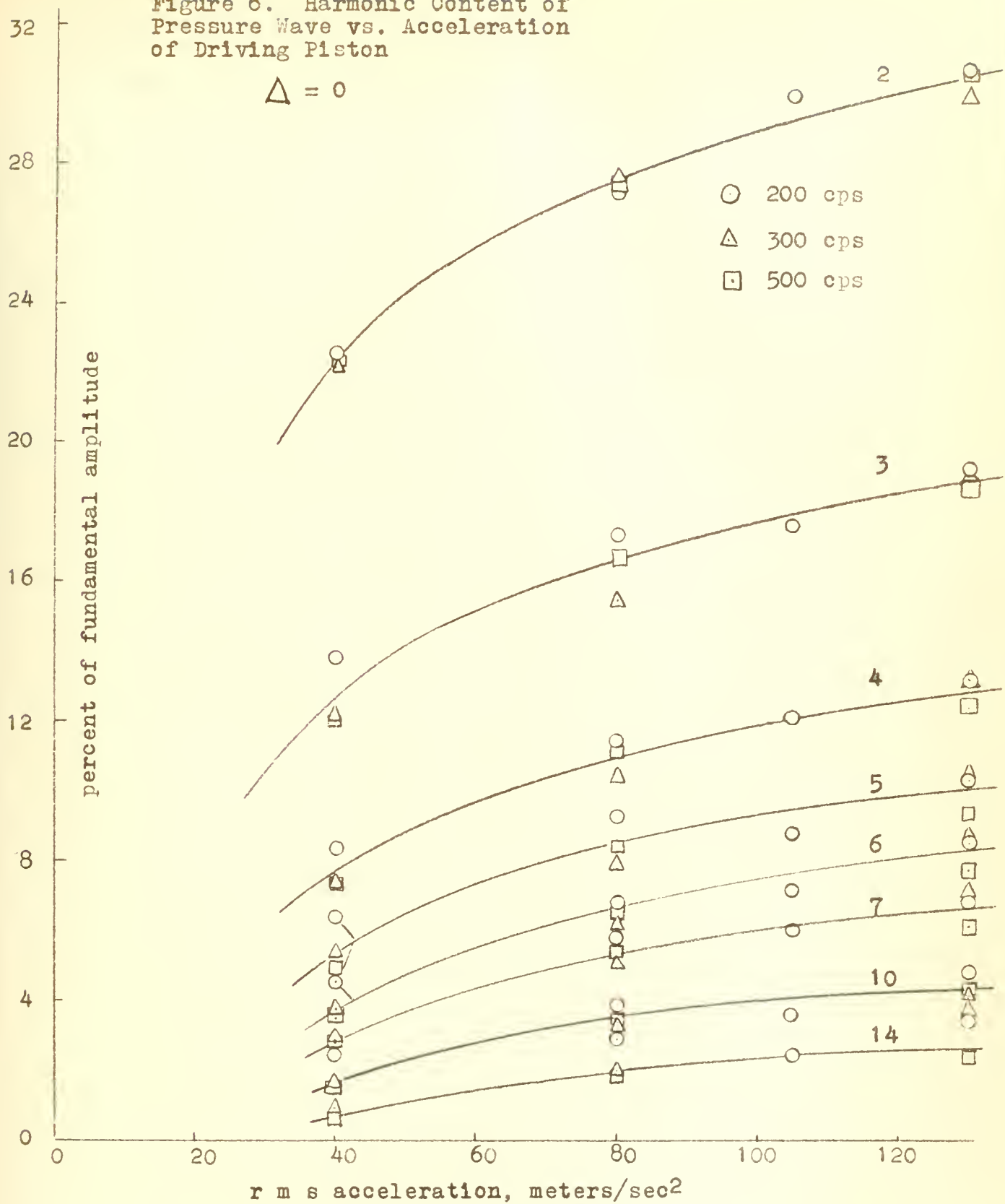
The numbers at the right indicate the order of harmonic.

24

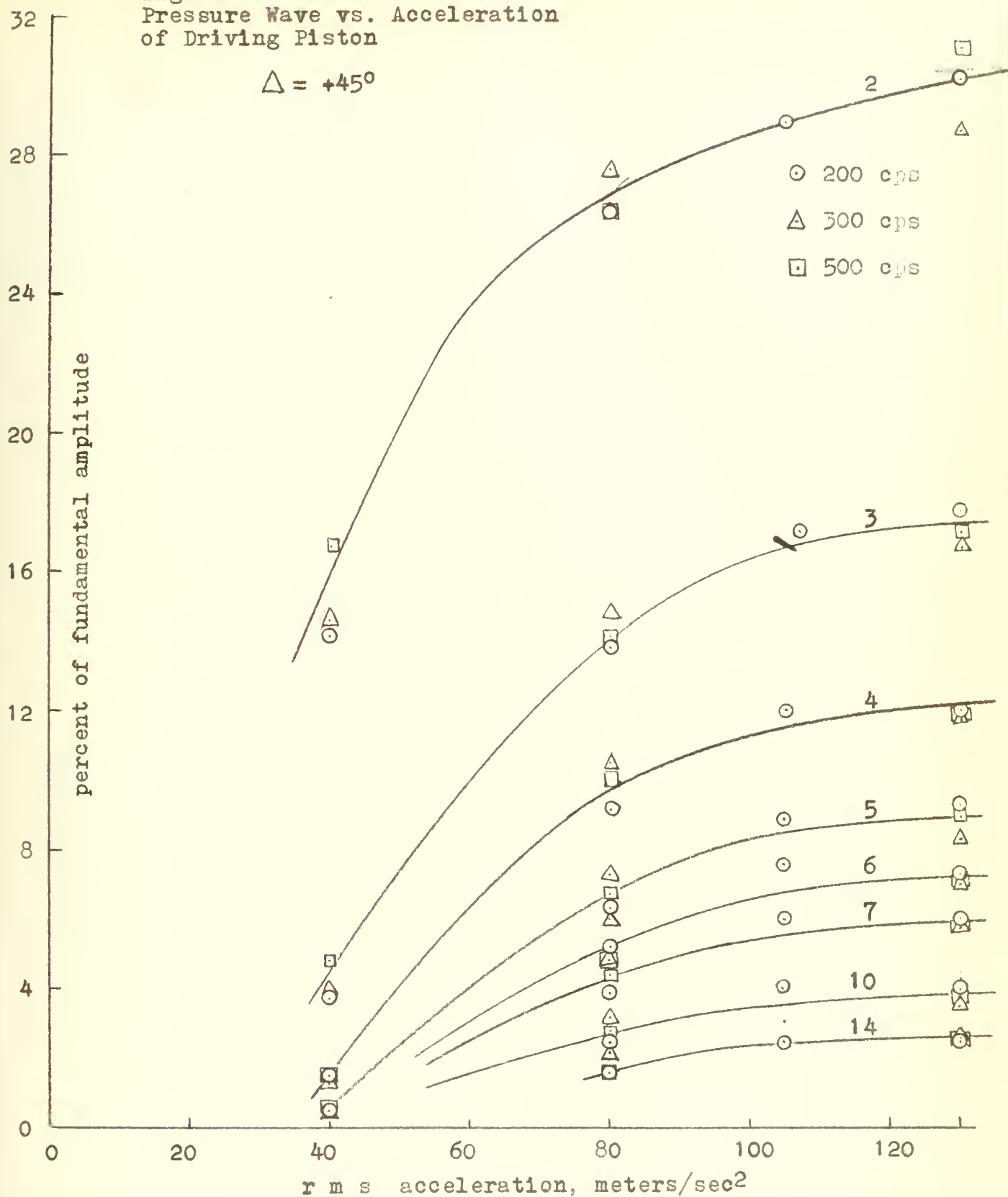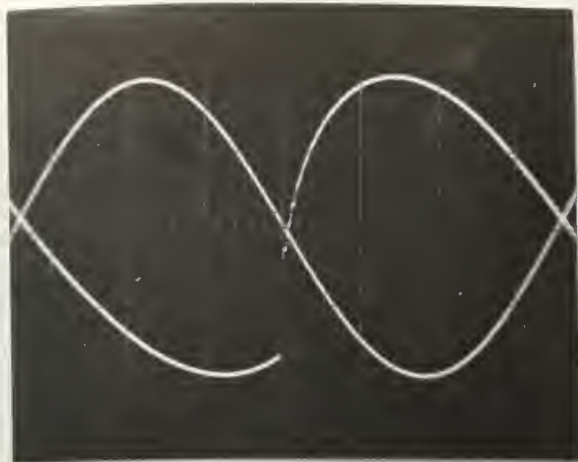Figure 7.   Harmonic Content of Pressure Wave vs. Acceleration of Driving Piston

The numbers at the right indicate the order of the harmonic.

25

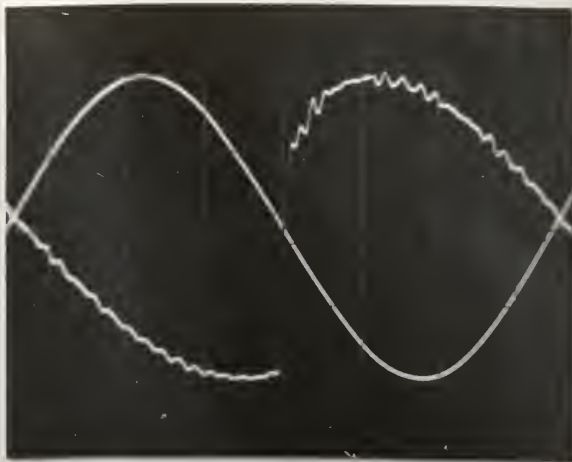Figure 8. Harmonic Content of Pressure Wave vs. Acceleration of Driving Piston

$\triangle = +45°$

2

⊙ 200 cps
△ 300 cps
⊡ 500 cps

3

4

5

6

7

10

14

percent of fundamental amplitude

r m s   acceleration, meters/sec^2
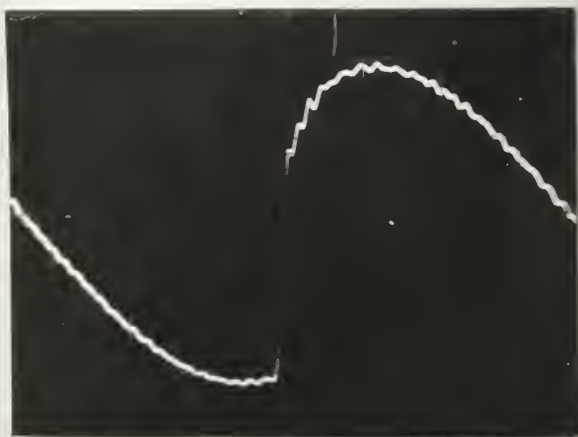
The numbers at the right indicate the order of the harmonic.
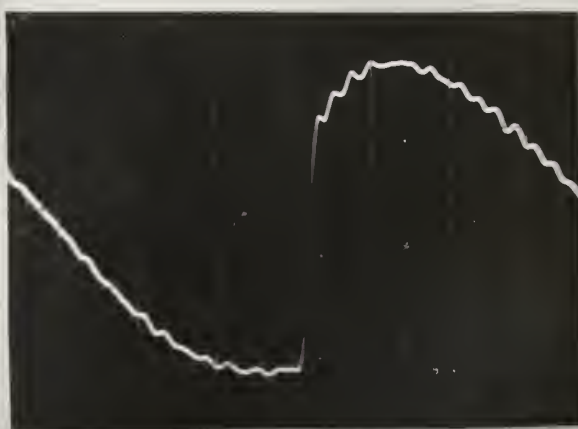
26

f = 100 cps

V = 6.9 cm/sec RMS

f = 300 cps

V = 6.9 cm/sec RMS

Figure 9.   Oscillograms of pressure-time waves at
different frequencies with the same piston
velocity.

f = 300 cps

$\mathcal{A}$ = 140 m/sec^2 RMS

f = 500 cps

$\mathcal{A}$ = 140 m/sec^2 RMS

Figure 10.   Oscillograms of pressure-time waves at
different frequencies with the same piston
acceleration.

The reader will note the remarkable similarity in the shape in the second case. The slight difference in height of the shockfront in the latter set may be due to the fact that microphone rise time assumes a relatively larger portion of the cycle in the higher frequency case. The small fluctuation superimposed on the basic waveform is due to the ringing of the microphone at the frequency of its mechanical resonance.

It must be pointed out that merely determining the harmonic amplitudes of the waves does not in itself give a complete picture of their construction, since the complete Fourier analysis is given by

$$P = A_0 + A_1 \cos(\omega t + \phi_1) + A_2 \cos(2\omega t + \phi_2) + \ldots$$

and no determination of the constants ϕ_1 was made in this experiment. Indeed, the fact that the visual appearance of the wave changes drastically with $\triangle$ while the curves of Figs. 6 through 8 change only slightly at larger amplitudes indicates that there must be a substantial change in the ϕ_1 as $\triangle$ is changed. The difficulty in determining the ϕ_1 is due to the asymmetry of the waves which prevents the selection of a satisfactory zero reference which will be common to all waves, regardless of $\triangle$. This same difficulty made it impossible to determine reasonable results for A_0; the computer program as written solved for this quantity, but since no common vertical zero reference was available for all waves analyzed, the numbers obtained were meaningless.

   c.   Variation of $\triangle$ with acceleration.

In the course of the investigation it was noticed that during a series of measurements which were intended to be at

constant $\triangle$, a change in the value of $\mathcal{A}$ was often accompanied by a change in the value of $\triangle$, even though the frequency was maintained constant. Therefore, it was decided to investigate this variation. A series of measurements was made which began with an initial setting of $\triangle$ at the maximum value of acceleration available on the shaker; $\triangle$ was then recorded as the acceleration was reduced. This procedure was carried out for seven different initial $\triangle$-settings for each of three different frequencies. The value of $\triangle$ measured at the maximum available value of $\mathcal{A}$ will be termed $\triangle_{max}$. For each frequency a similar set of curves were obtained, included herein as Figs. 11 through 13. In general, if the $\triangle_{max}$ was positive, the value of $\triangle$ became more positive as $\mathcal{A}$ was reduced; similarly, a negative $\triangle_{max}$ became more negative. However, it was interesting to note that a $\triangle_{max}$ of zero did not remain zero with decreasing $\mathcal{A}$, but rather became positive. In addition, there was one negative value of $\triangle_{max}$, which was different for each frequency checked, for which $\triangle$ remained constant as $\mathcal{A}$ was reduced. The measured positions of these constant-$\triangle$ lines are shown as dashed horizontal lines in Figs. 11 through 13.

The close similarity of these sets of curves for the different frequencies again demonstrates the existence of $\mathcal{A}$ as a similarity parameter.

d. Shock Strength

The most striking feature of the nonlinear wave of finite amplitude is the appearance of a sharp pressure discontinuity as the shock passes the recording device. When the microphone is installed in the rigid tube-end as it was in this experiment,

a double-amplitude shock is observed, as the incident and re-
flected shocks are superimposed.

Shock strength variation was studied over nine different
values of $\triangle$ at each of six frequencies, while varying $\mathcal{A}$ over
its available range. The strength of the shock was measured by
recording the height of the shock as observed on the calibrated
oscilloscope face. From the pressure calibration of the micro-
phone it was possible to obtain shock strength as a function
of f, $\triangle$, and $\mathcal{A}$. A series of quasi-linear curves were ob-
tained which are included herein as Figs. 14 through 17.

Although the response of the microphone is not perfectly
constant over all frequencies, it is sufficiently flat over many
harmonics that the single figure of -85 db re 1 volt per dyne
per square centimeter can be used to compute pressure from volt-
age output. It was also necessary, of course, to halve the
pressure obtained because of the double-amplitude nature of
the wave as measured, in order to compute incident shock over-
pressure.

The qualitative characteristics of the shock variation are
similar for all frequencies studied. At any $\triangle$-setting, as
the acceleration is reduced, the shock diminishes in strength
and finally disappears. At the time that the shock disappears,
the wave retains the form of a distorted sine wave, and it is
not until $\mathcal{A}$ is much further reduced that the pure sinusoidal
character of the acoustic wave is assumed. The value of $\mathcal{A}$ for
which the shock disappears is not the same for all frequencies,
being about twice as great for 1000 cycles as for 100 cycles.

In order to test the previous conclusion that the shape of

30

the wave can be predicted from piston acceleration, each value
of shock strength data was divided by the amplitude of the fun-
damental component of the wave measured under the same condi-
tions. This was done to take into account the varying strengths
encountered at the different frequencies. The points resulting
from this calculation were then plotted against $\mathcal{A}$, giving the
set of curves shown in Figs. 18, 19 and 20. These curves essen-
tially support the contention that $\mathcal{A}$ is a similarity parameter,
since for every Δ, the points obtained by the above operation
fall, within the limits of experimental precision, along a sin-
gle line, regardless of frequency.

By a careful study of the work of Saenger and Hudson [8],
one may conclude that the existence of the similarity parameter
$\mathcal{A}$ for shock strength can be inferred at least qualitatively
from their predictions. Their development results in the ex-
pression

$$\frac{\Delta P}{P_0} = \frac{X}{\pi \Phi L} \frac{6\gamma}{(\gamma - 1)B + \Phi} \sin(\Delta - \psi') \quad (6)$$

where Φ is the dissipation parameter for viscous losses and
B is a parameter derived from the proportionality constant in
Newton's law of cooling. According to Saenger and Hudson,
agreement between experiment and theory could not be obtained
unless empirical values for B and Φ were used. These were
such that $\Phi \gg B$. Hence (6) becomes, approximately

$$\frac{\Delta P}{P_0} = \frac{X}{\pi \Phi L} \frac{6\gamma}{\Phi} \sin(\Delta - \Phi) \quad (7)$$

In a theoretical investigation of the propagation of sound
in a closed tube, Rayleigh [9] derived an expression for the
tangential force per unit area on the wall by viscous effects

31

of air moving at unit velocity. Expressed per unit length of
tube, this is

$$F = \pi a \sqrt{2 \omega \rho \mu}$$

To obtain the Φ of Saenger and Hudson from this, F must be di-
vided by the mass of the gas per unit length, $\rho \pi a^2$ (because of
their use of the Lagrangian notation), and by ω (because of
their use of nondimensionalized quantities), giving

$$\Phi = \frac{1}{a} \sqrt{\frac{2 \nu}{\omega}}$$

Numerically, this does not agree with the empirical results of
Saenger and Hudson. Nevertheless, the important consideration
here is the frequency dependence; it is noted that Φ is pro-
portional to $f^{-\frac{1}{2}}$. Hence, a further approximation of (7) is

$$\frac{\Delta P}{P_0} = \frac{3 \gamma a^2}{\pi \nu} \left(\frac{\omega X}{L} \right) \sin(\Delta - \Phi) \tag{8}$$

Applying (5), equation (8) may be written

$$\frac{\Delta P}{P_0} = \frac{3 \gamma a^2}{\pi^2 \nu c} \left(\omega V \right) \sin(\Delta - \Phi)$$

which becomes

$$\frac{\Delta P}{P_0} = \frac{3 \gamma a^2}{\sqrt{2} \pi^2 \nu c} A \sin(\Delta - \Phi) \tag{9}$$

The behavior of P_0 is not well-known, either as a func-
tion of Δ or of frequency. More investigation must be done to
reconcile if possible the facts that, although ΔP is near maxi-
mum at $\Delta = 0$ (see Fig. 21), the right member of (9) is near
minimum at the same value of Δ.

    e. Phase Diagrams

    In taking the data for the shock strength variation, the
minimum value of A for which a shock could be measured was re-
corded at each of the resonant frequencies studied. This mini-

mum occurred at or near the $\Delta = 0$ frequency for the tube.
Plotted against frequency in Fig. 24, these points furnish a
phase diagram which separates the $\mathcal{A}$-f plane into regions in
which shocks occur and do not occur. These are labelled "Shock
Region" and "Acoustic Region," respectively. A similar phase
diagram, prepared for displacement amplitudes instead of ac-
celeration, is included as Fig. 25.

If $\mathcal{A}$ were in fact a perfect similarity parameter, then
the curve of Fig. 24 would be a horizontal straight line.
Since it is not, this shows that although the concept of $\mathcal{A}$
as a similarity parameter for shock waves is valid over a large
range of piston amplitudes, it breaks down in the limit of very
small amplitudes.

The theory of Saenger and Hudson predicts the minimum
piston amplitudes at which shocks may occur. A calculation
based on this theory for $f_1 = 100$ cycles per second predicts a
minimum displacement amplitude for the shock region which is
about thirty times that obtained for the 100 cycle point on
Fig. 25.

f.   Check of Correlation Between "Acoustic" and "Shock"
Resonance Frequencies.

Although it has long been known that the production of
periodic shock waves in a closed tube occurs at frequencies
which are very near the acoustic resonant frequencies of the
tube, the writer knows of no instance where it has previously
been verified that these frequencies are in fact the same. In-
deed, the difference in magnitude of X for the two cases, as
well as the more complicated theory applicable in the finite

amplitude case, leads one to expect that they might conceivably be slightly different.

A check on these frequencies was a simple matter with the variable-amplitude driver used. First, the frequency-tubelength combination was adjusted so that $\triangle$ was exactly zero with the shaker operating at maximum acceleration. Then $\mathcal{A}$ was reduced using the GAIN control on the shaker until the output of the microphone was near its minimum detectable value. The sinusoidal appearance of the microphone output assured that this level was well within the acoustic domain. During the process of reducing the gain control, the frequency counter indicated that frequency was maintained constant to the nearest tenth of a cycle per second.

Finally, the microphone output was measured on an a. c. voltmeter. Several measurements were made following this procedure, at frequencies of 100, 300, 500 and 100 cycles. In every case, the voltmeter was found to be sharply peaked when the final measurement was made, and the slightest change in frequency in either direction caused an abrupt drop in the sound pressure as measured by the microphone.

From this it is concluded that the acoustic resonance frequency of a tube and the $\triangle_{\max} = 0$ condition for periodic shock waves occur at exactly the same frequency, within the limits of experimental precision.

(In view of the shift in the $\triangle_{\max} = 0$ lines of Figs. 11, 12 and 13 as $\mathcal{A}$ is decreased, the results are especially interesting. Examining the curves, one might suspect that the values of $\triangle_{\max}$ for which $\frac{d\triangle}{d\mathcal{A}} = 0$, represented by the dashed

lines, rather than the $\triangle_{max} = 0$ condition would be found to be the determining factor for the acoustic resonance frequencies, but this is definitely not the case.)

g. Variation of $\triangle_{max}$ with frequency.

During several series of measurements, the variation of $\triangle_{max}$ with frequency was studied. As could be anticipated, it was found that the change in frequency necessary to produce a given change in $\triangle_{max}$ was greater as frequency was increased. The frequency at which the tube is resonant to the fundamental will be denoted by f_1; then the change necessary to produce a given $\triangle_{max}$ is expressed by the quantity $f - f_1$. When $f - f_1$ was plotted against $\triangle_{max}$, a family of monotonically decreasing curves was obtained (Fig. 22). Since all curves are of similar shape, a parameter was sought by which the variation with respect to frequency could be quantitatively determined. After a short exploratory process, it was discovered that the quantity $(f - f_1)/f_1^{\frac{1}{2}}$ is the same function of $\triangle_{max}$, for all f_1. This result is shown in Fig. 23.

h. Thickness of Shock Wave.

All oscillograms taken show the shock wave as an abrupt rise in pressure which takes place in a time so short that it could not be measured by the Altec microphone with its rise time of about sixty microseconds. In an effort to determine more about the structure of the shock, a new microphone was constructed. The pressure-sensitive element of the new microphone was a barium titanate wafer with a two-megacycle resonant frequency. The wafer was mounted in a brass plug fitted in the same manner as the plug which held the Altec microphone.

Using a high-gain preamplifier with the Tektronix oscilloscope, the output of the barium titanate crystal in response to periodic shock waves could be observed. Although there was a considerable amount of ringing (due to the mechanical resonance of the crystal) superimposed on the familiar profile of the shock wave, the pressure jump of the shock wave was clearly visible, and its rise time could be measured. Careful measurements at several frequencies using the expanded scale of the oscilloscope gave rise times of the order of one-half microsecond. From this it is concluded that the times measured were those of the instrument and not of the shock wave, and that the shock pressure jump must be completed in less than one-half microsecond. The shock thickness, then, must be less than 0.2 millimeter, which is about the distance a sound wave travels in one-half microsecond.
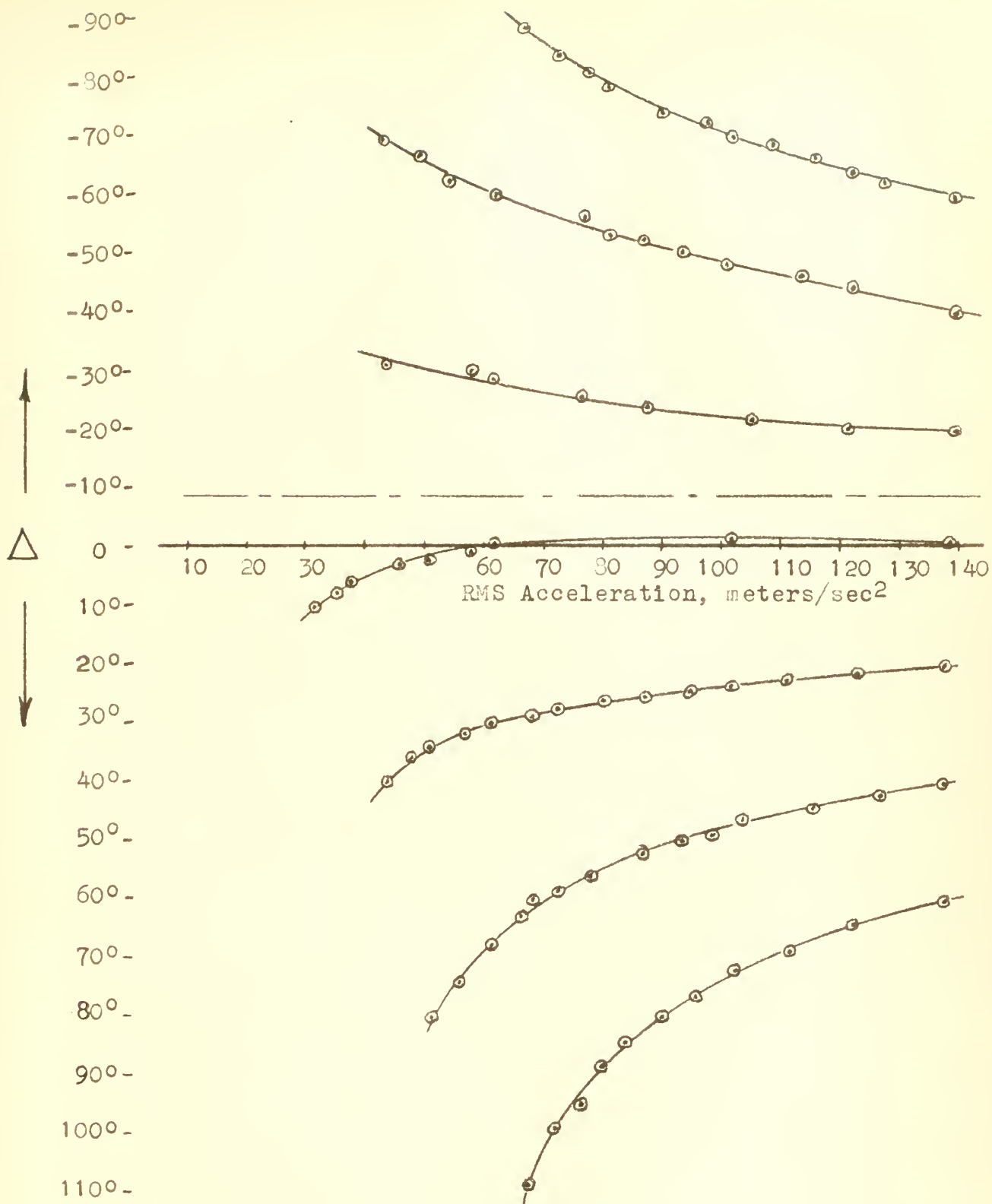
Figure 11. Variation of $\triangle$ as $\mathcal{A}$ is reduced. $f_1 = 100$ cps.

Data was taken by setting initial values of $\triangle_{max}$
and recording $\triangle$ as $\mathcal{A}$ was reduced.

37
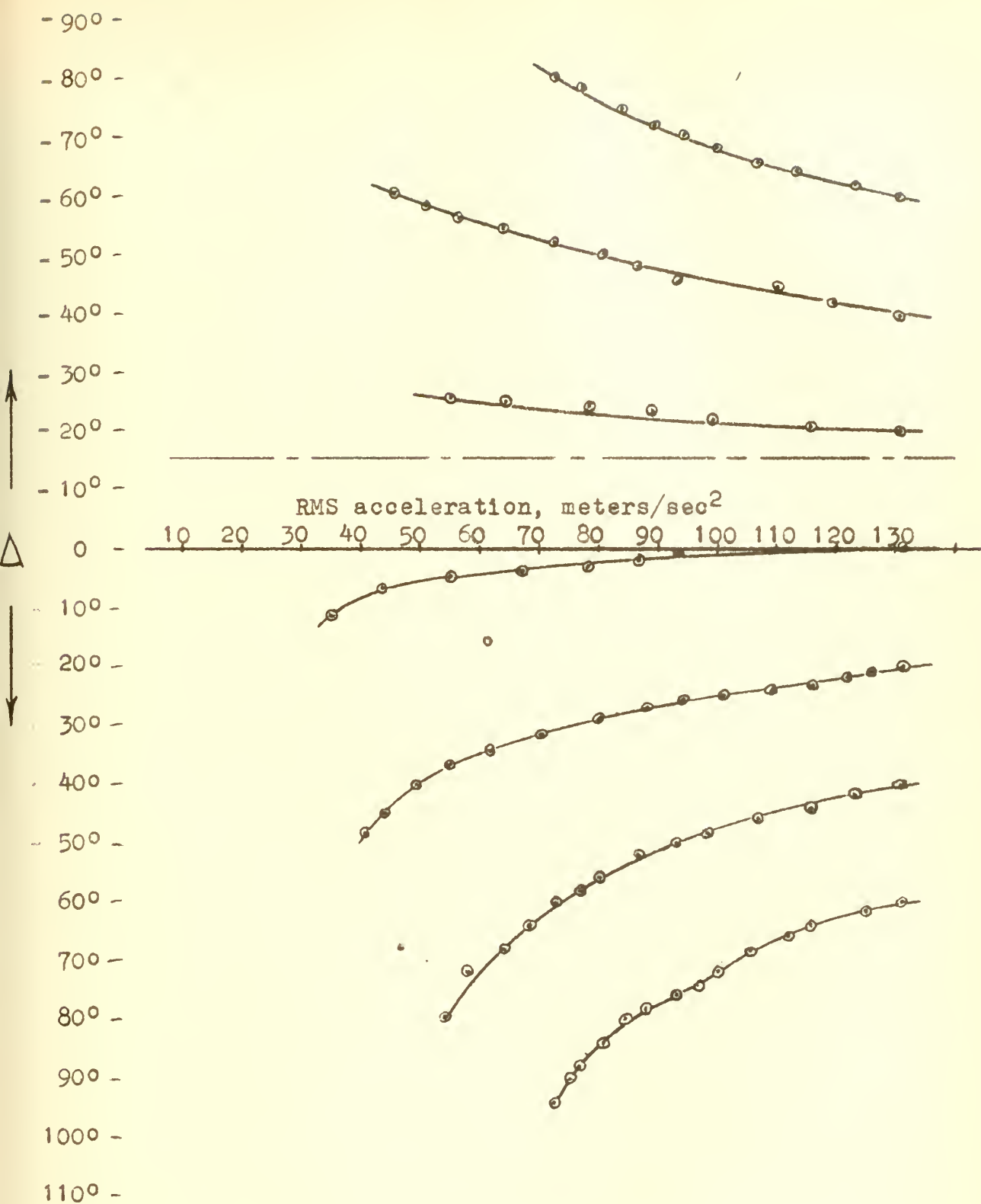
Figure 12. Variation of Δ as $\mathcal{A}$ is reduced. f_1 = 200 cps.

Data was taken by setting initial values of Δ_{max} and recording Δ as $\mathcal{A}$ was reduced.
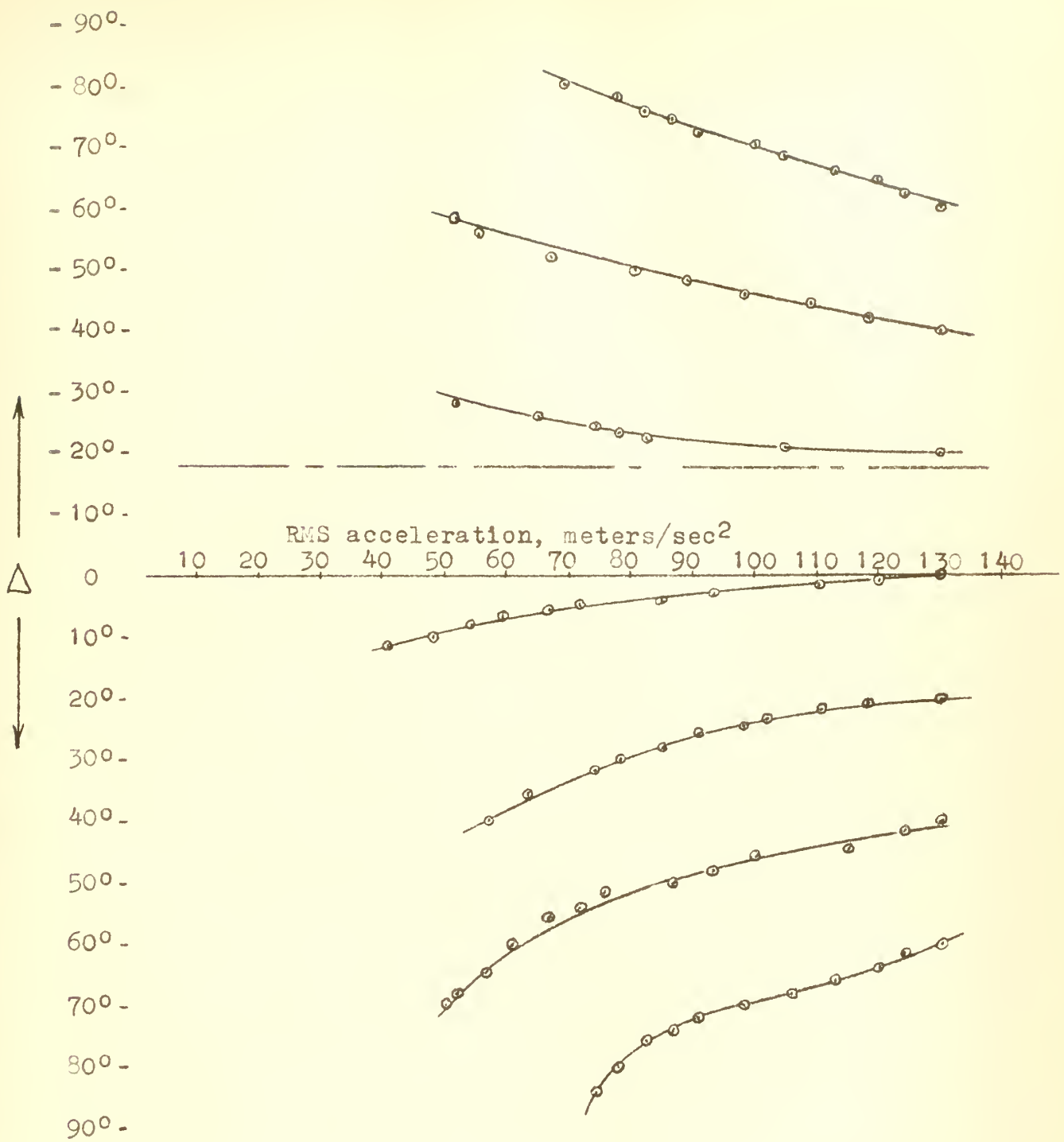
Figure 13. Variation of $\triangle$ as $\mathcal{A}$ is reduced. $f_1 = 300$ cps.

Data was taken by setting initial values of $\triangle_{max}$ and recording $\triangle$ as $\mathcal{A}$ was reduced.
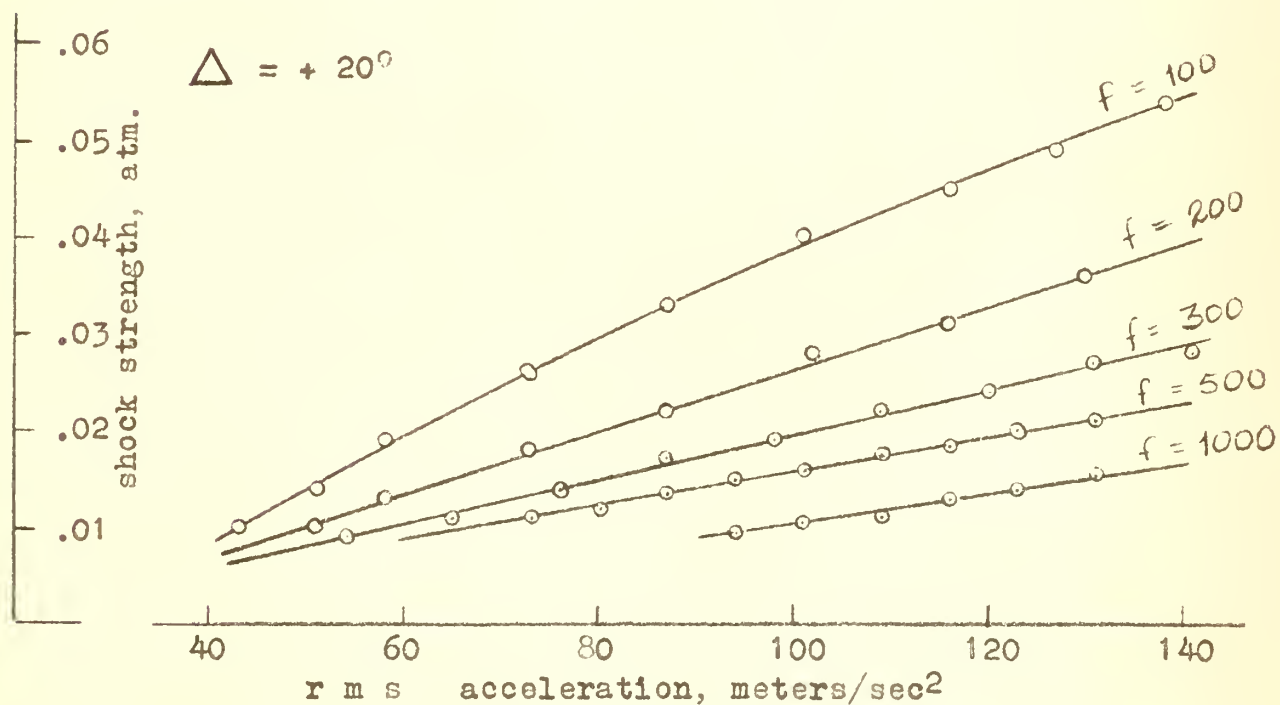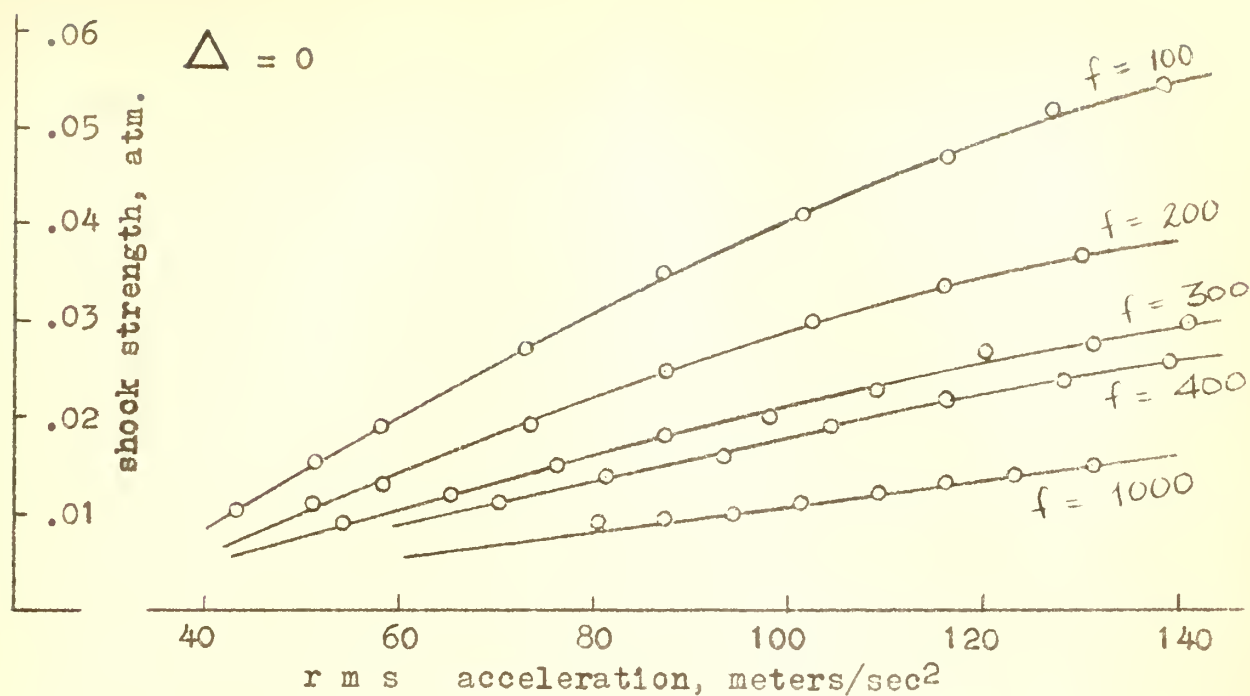
Figure 14. Shock Strength versus A.
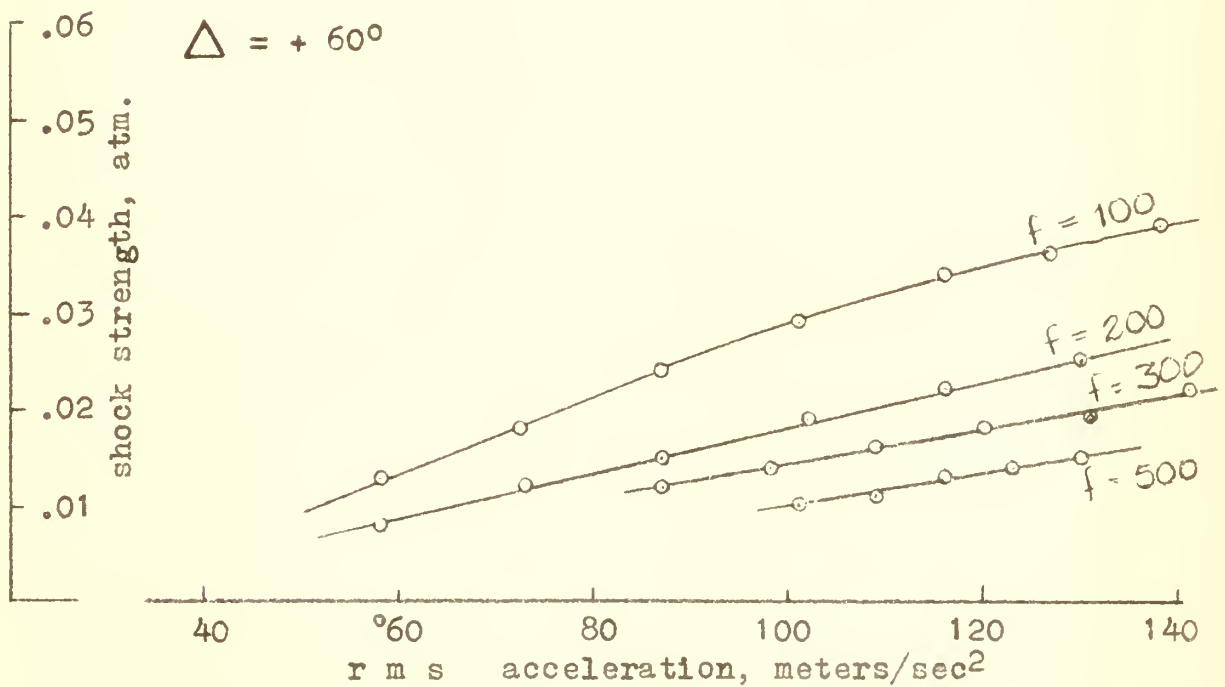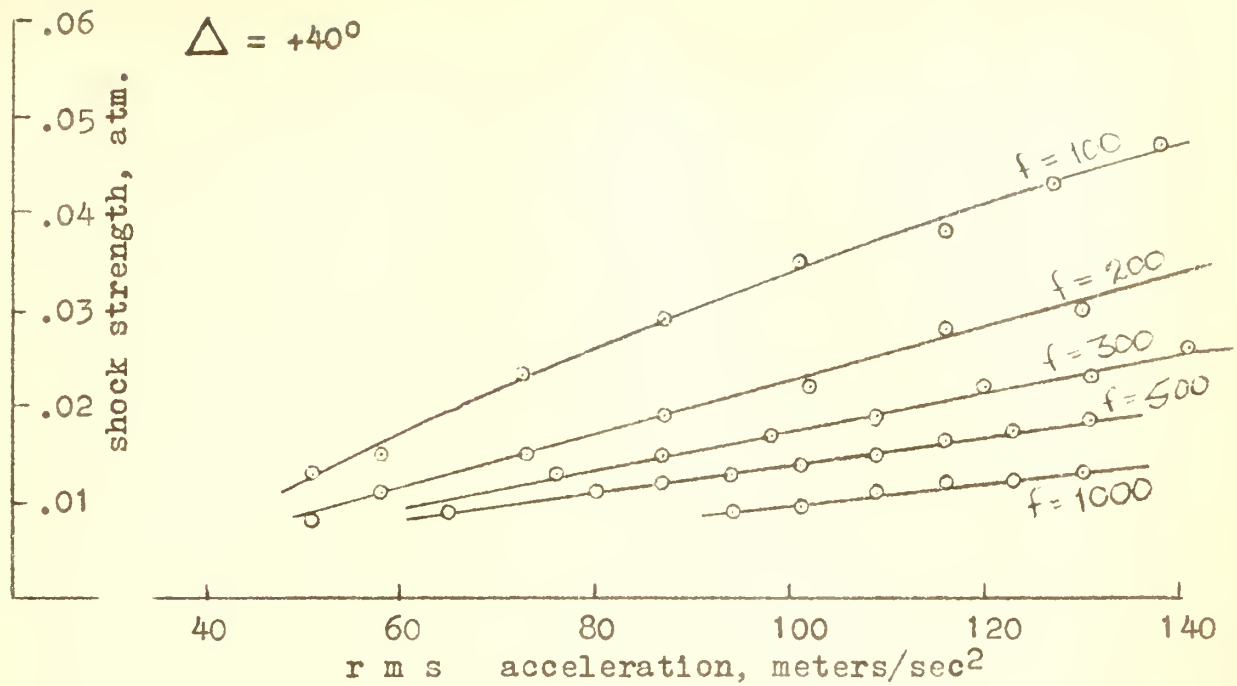
Figure 15. Shock Strength versus $\mathcal{A}$.
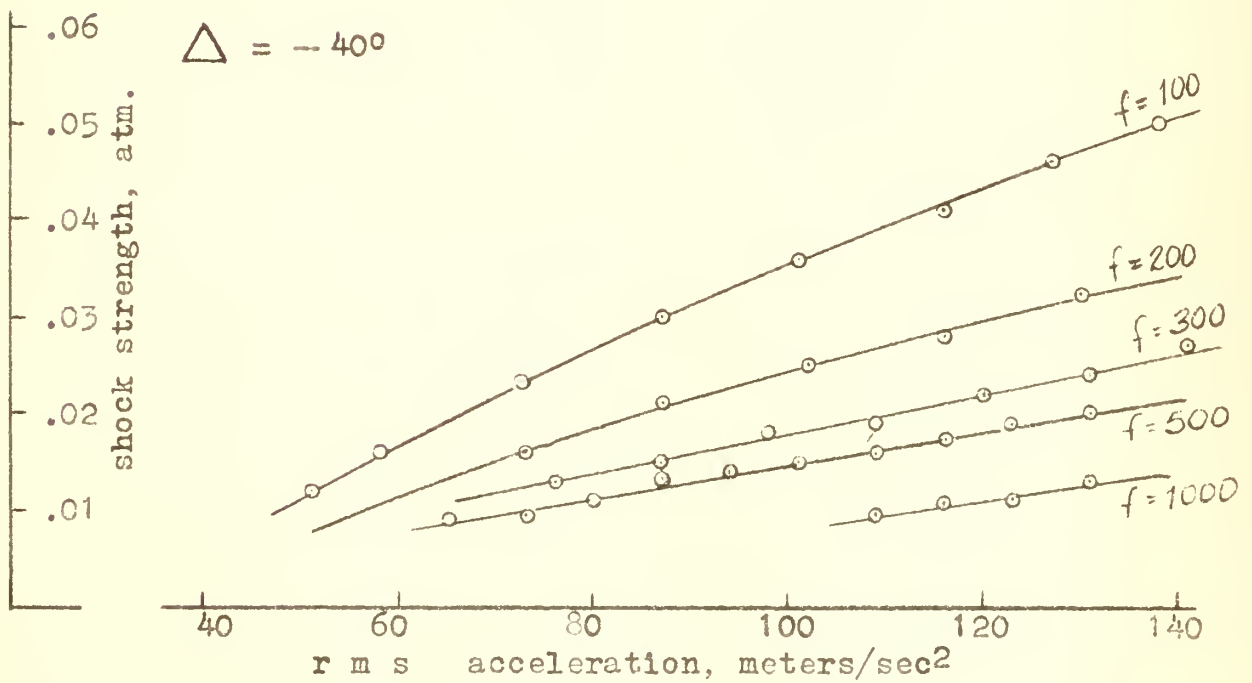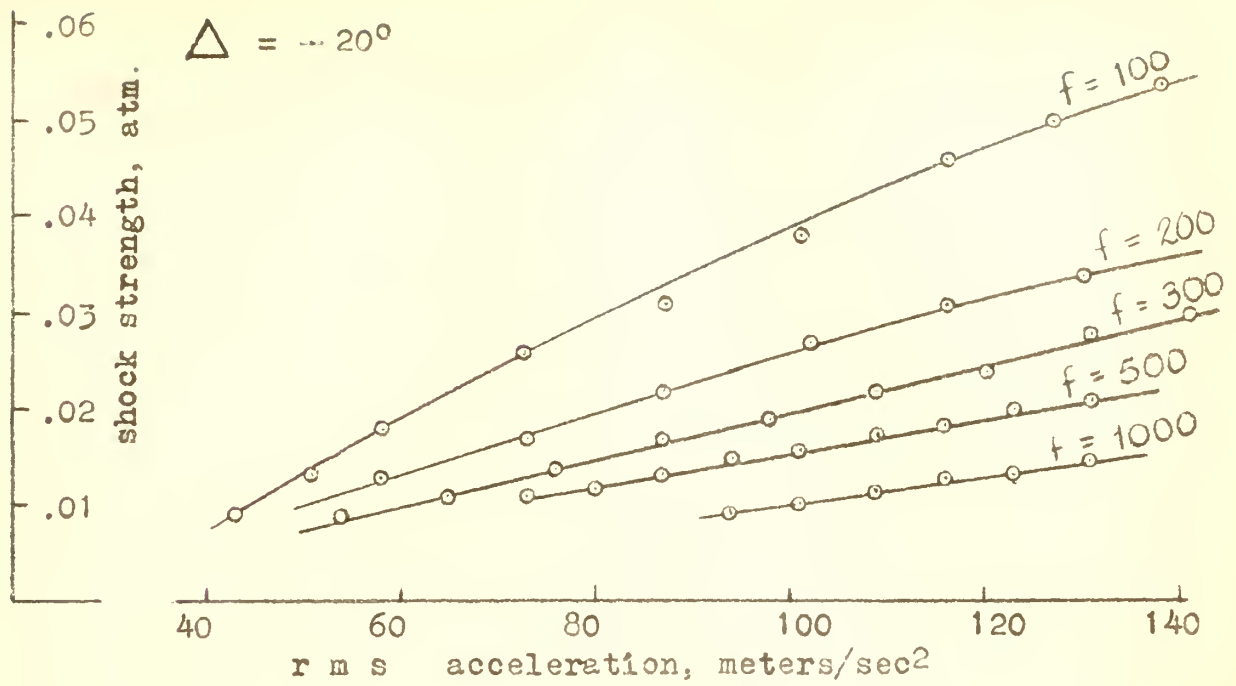
Figure 16. Shock Strength versus A.

42
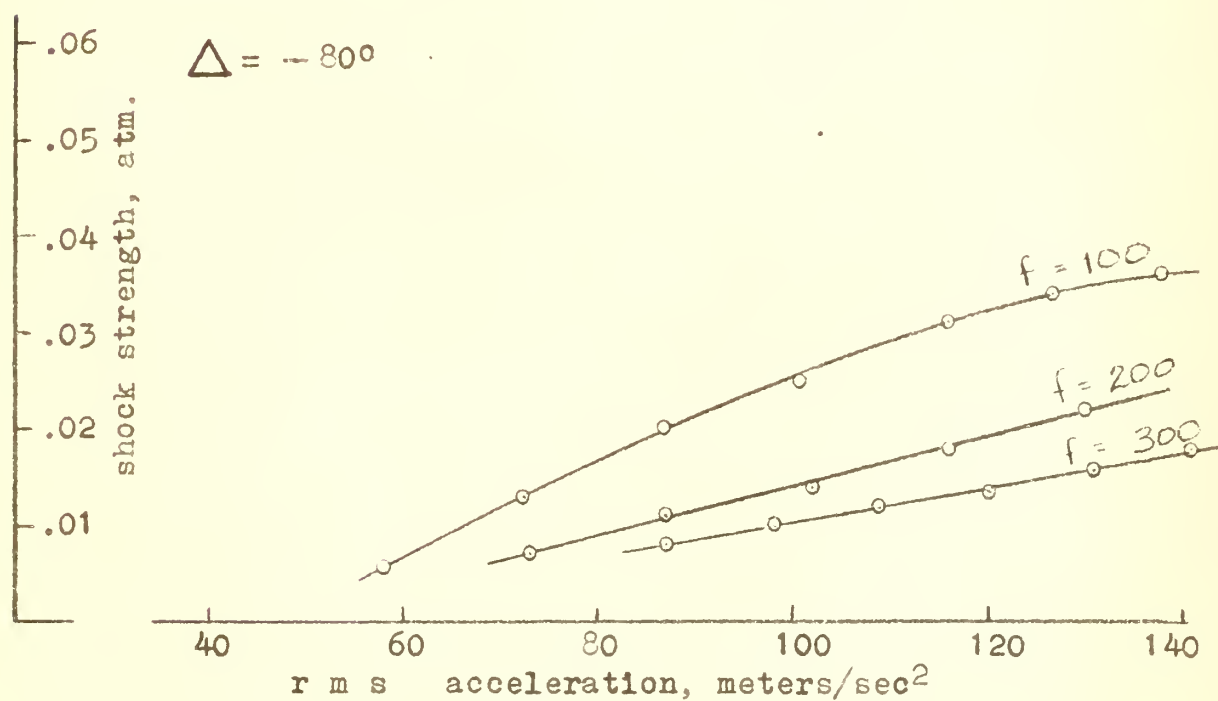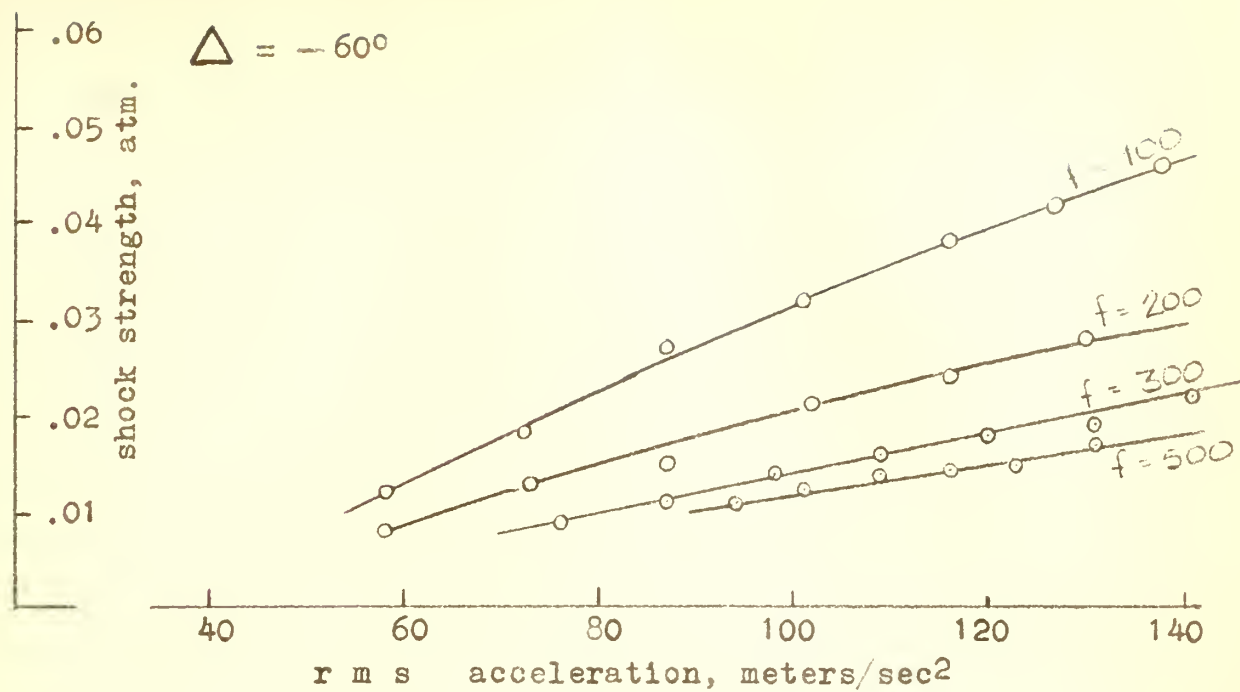
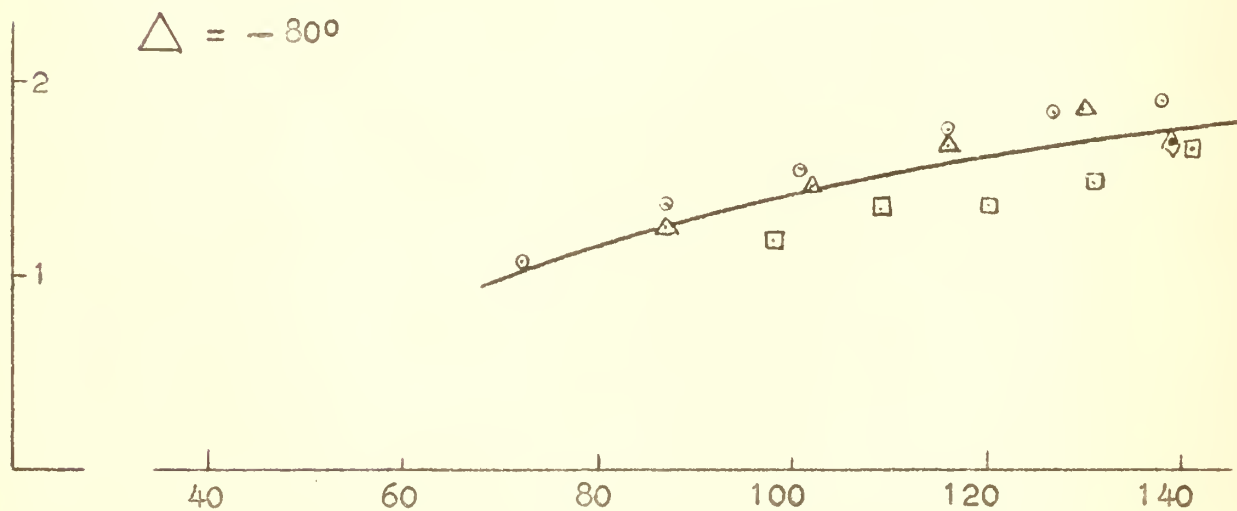Figure 17.   Shock Strength versus A.

43
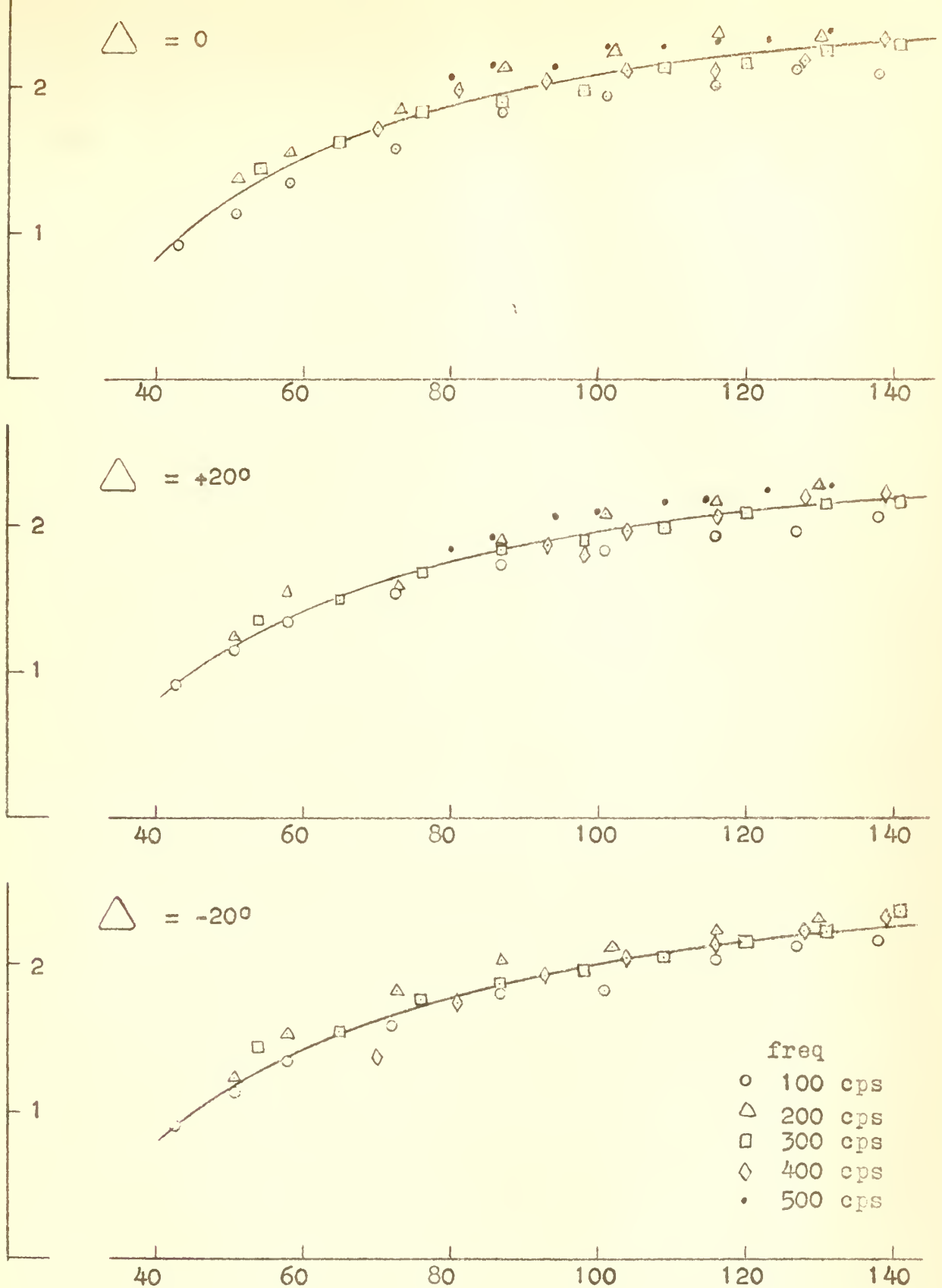
Figure 18.   Normalized Shock Strength versus $\triangle$.

Vertical scale is $\triangle P$ in volts (as measured on oscilloscope) divided by fundamental of wave in rms volts.

44
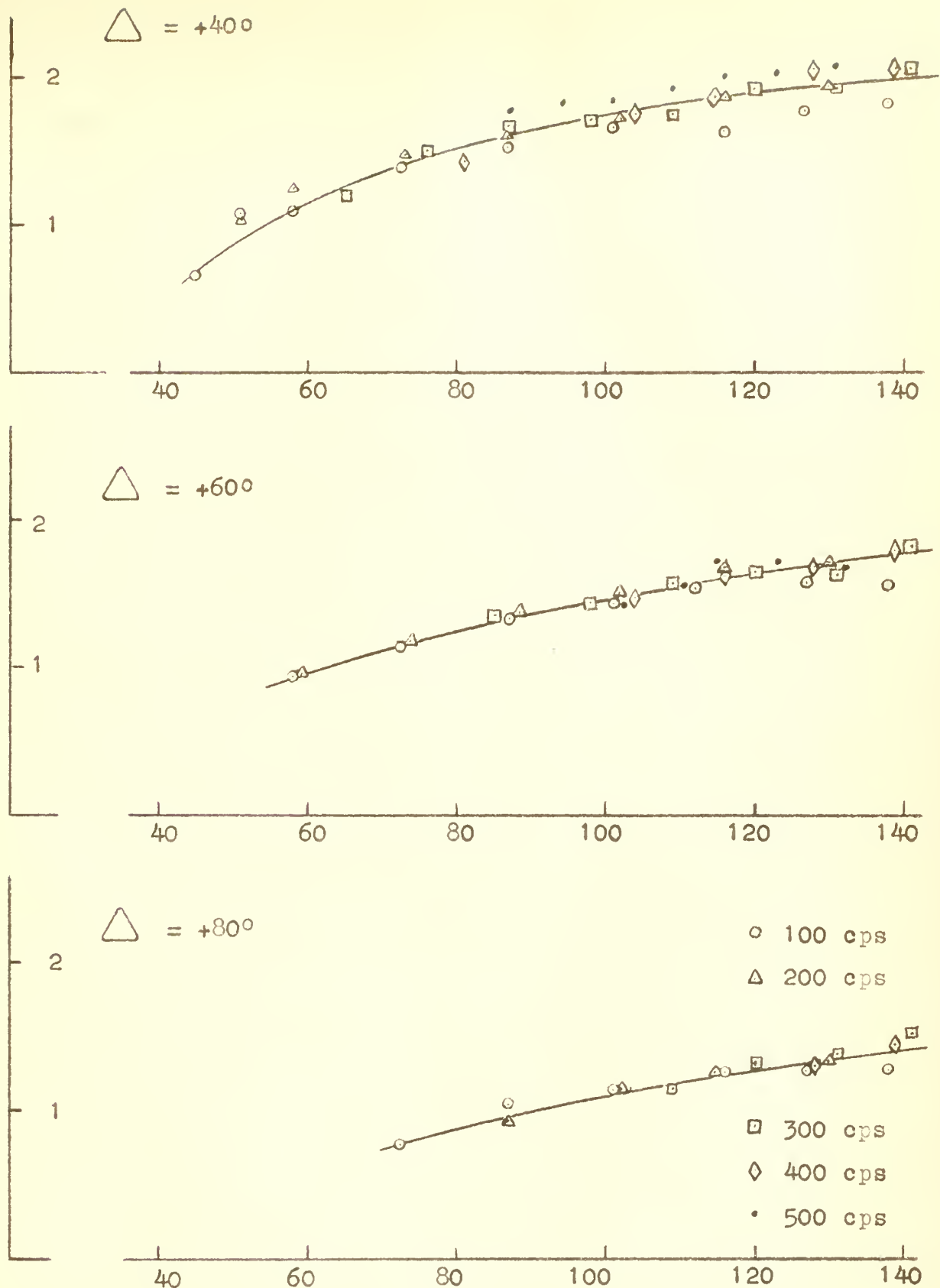
Figure 19. Normalized Shock Strength versus A.

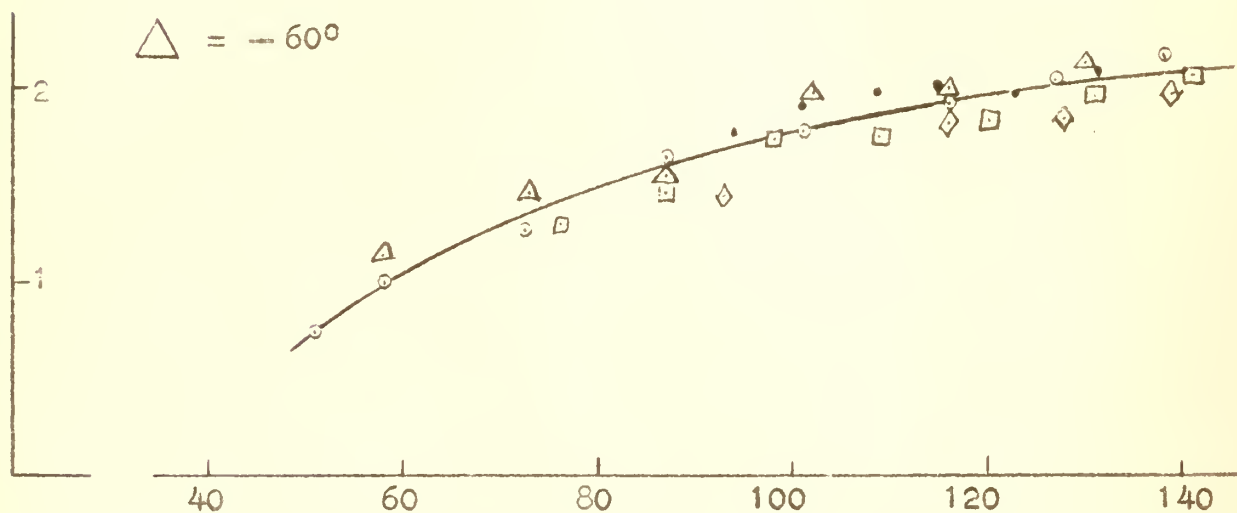Vertical scale is ΔP in volts (as measured on oscilloscope) divided by fundamental of wave in rms volts.
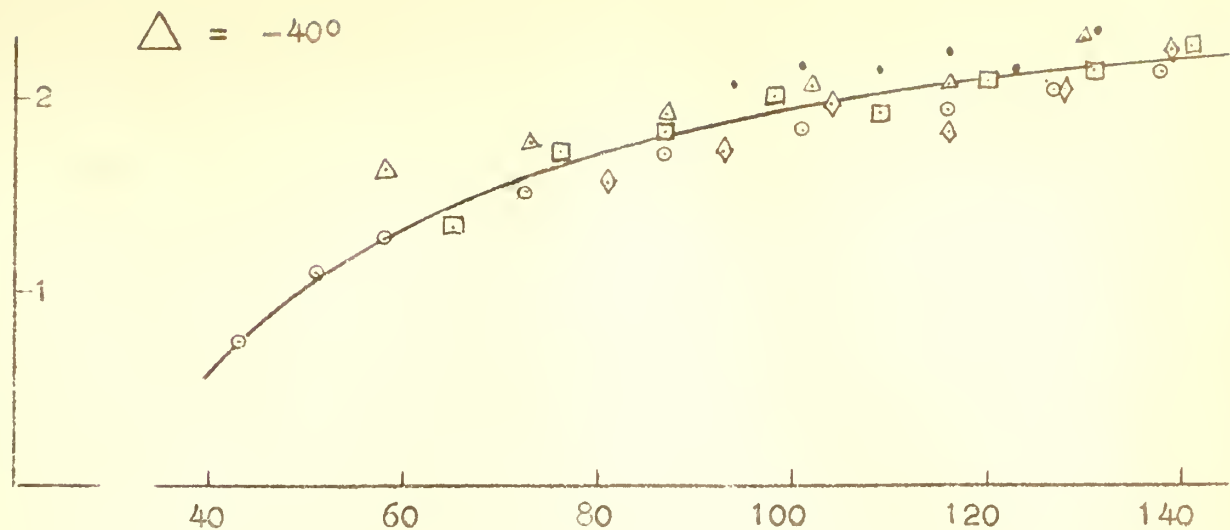
45

Figure 20. Normalized Shock Strength versus A.

Vertical scale is ΔP in volts (as measured on oscilloscope) divided by fundamental of wave in rms volts.
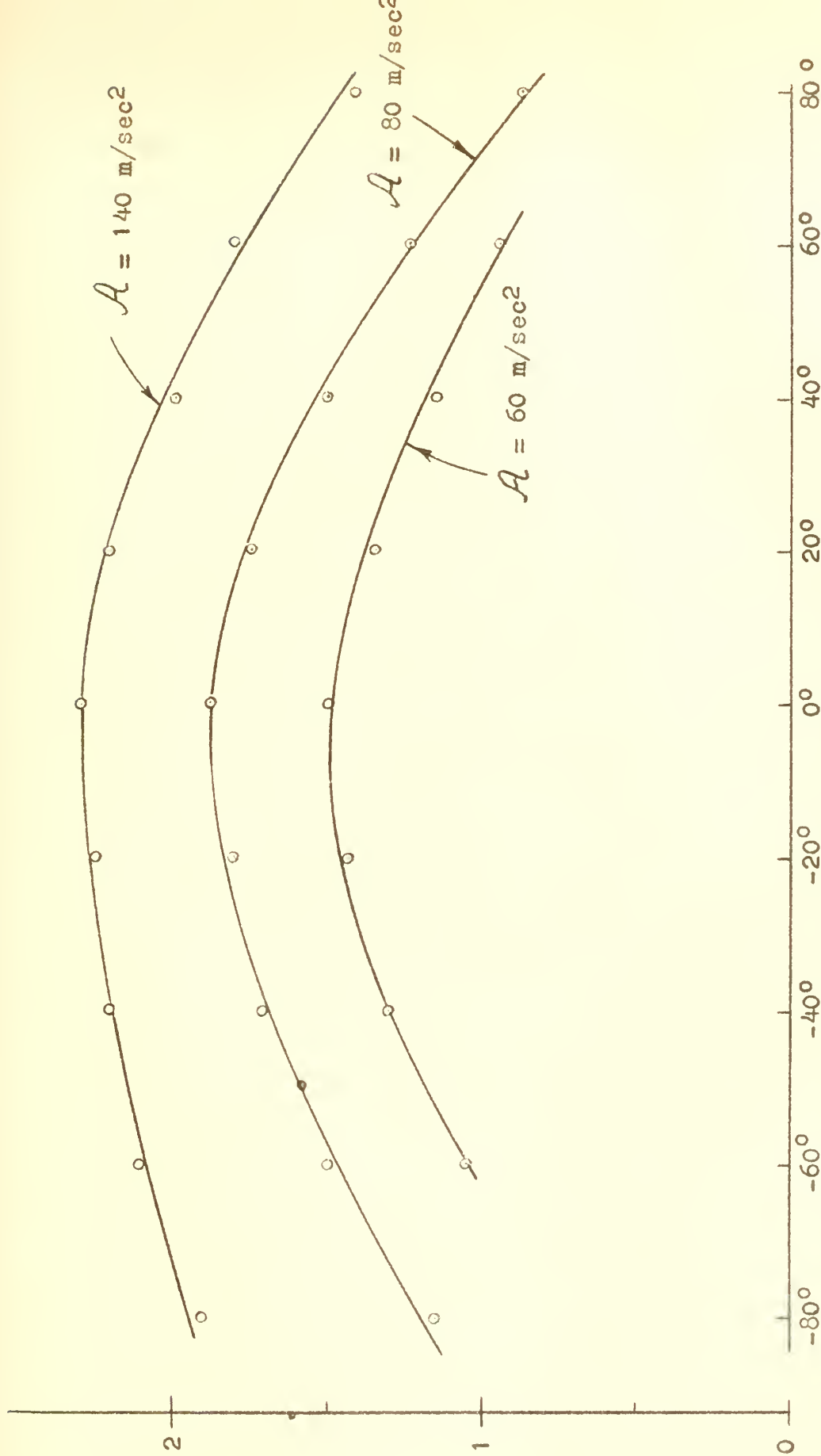
46

Figure 21. Normalized Shock Strength versus Δ.

Vertical scale is ΔP in volts (as measured on
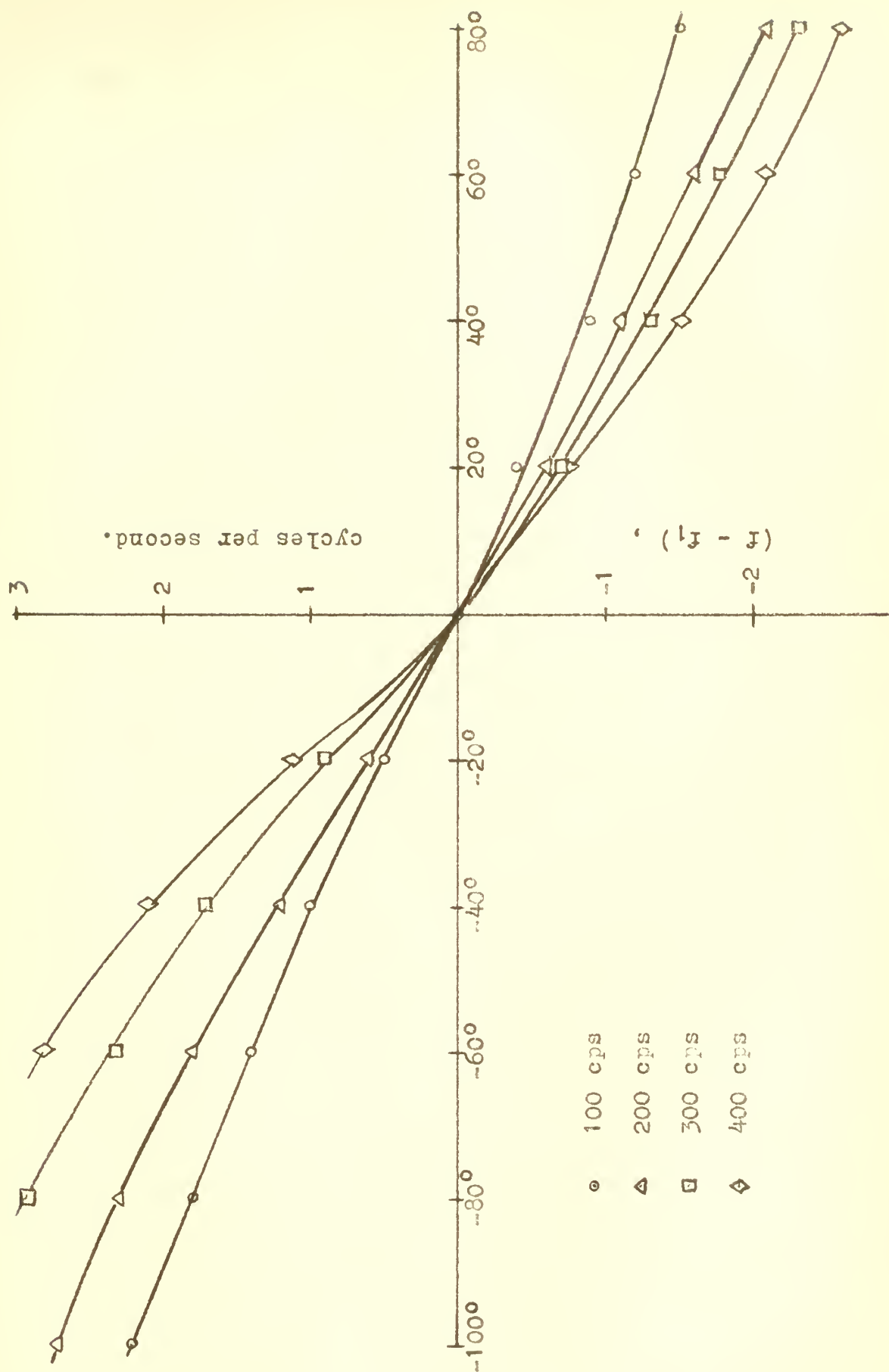oscilloscope) divided by fundamental of wave in rms volts.
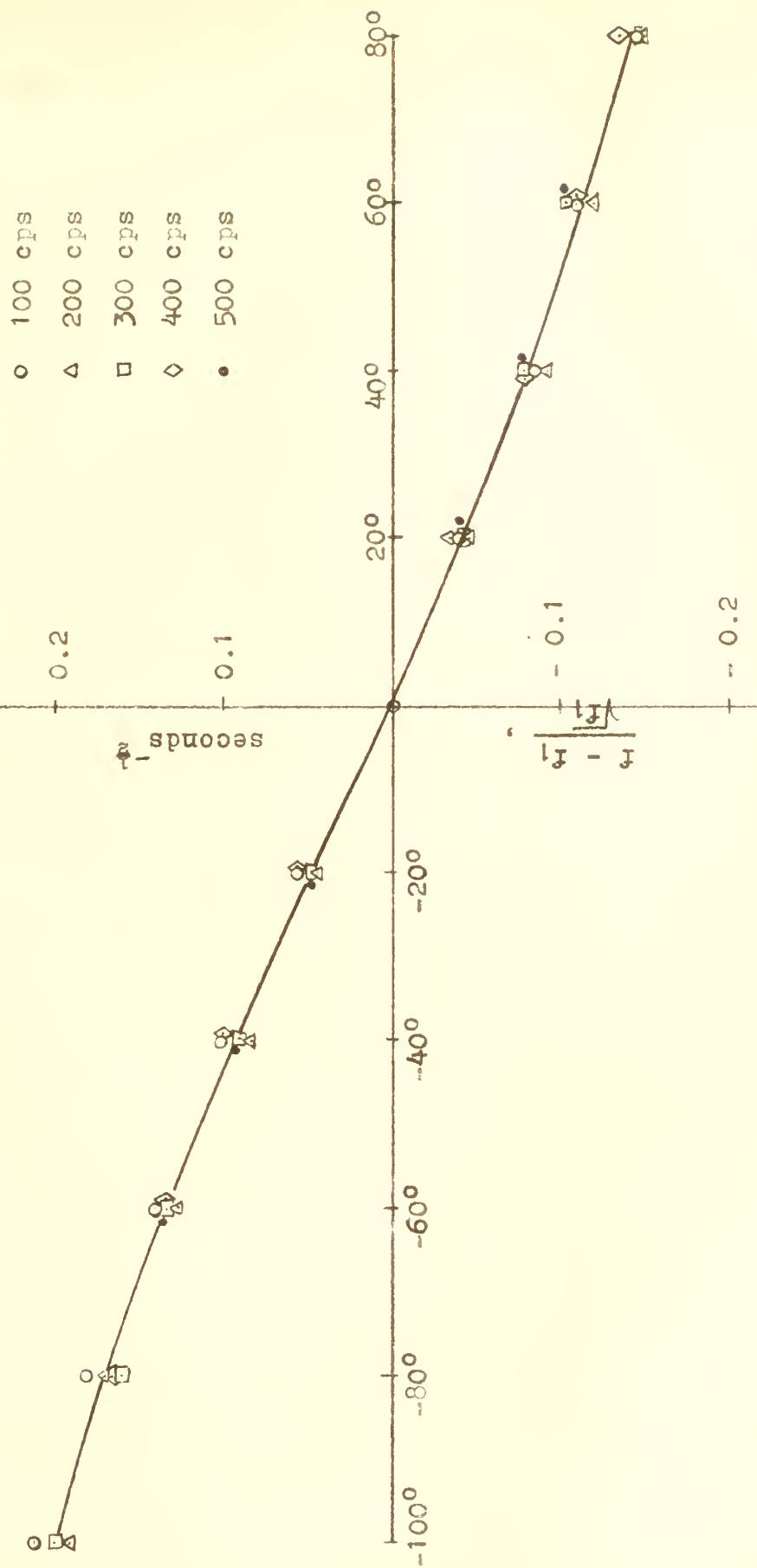
47

Figure 22. Variation of $(f - f_1)$ with $\triangle_{max}$.

48

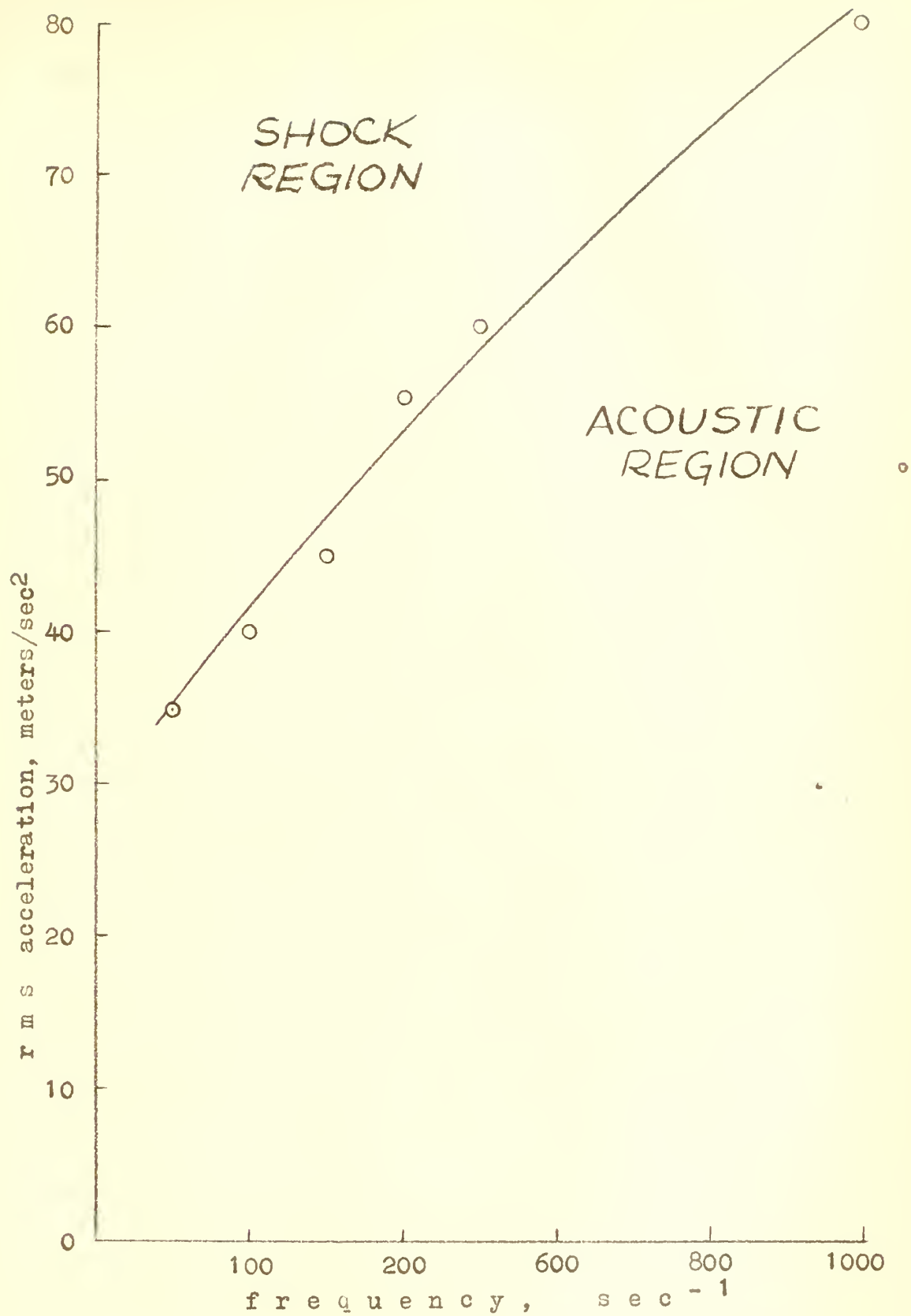Figure 23. $\left[\dfrac{f - f_1}{\sqrt{f_1}}\right]$ as a function of $\triangle_{max}$.

49

Figure 24.   Phase Diagram Showing Threshold Piston
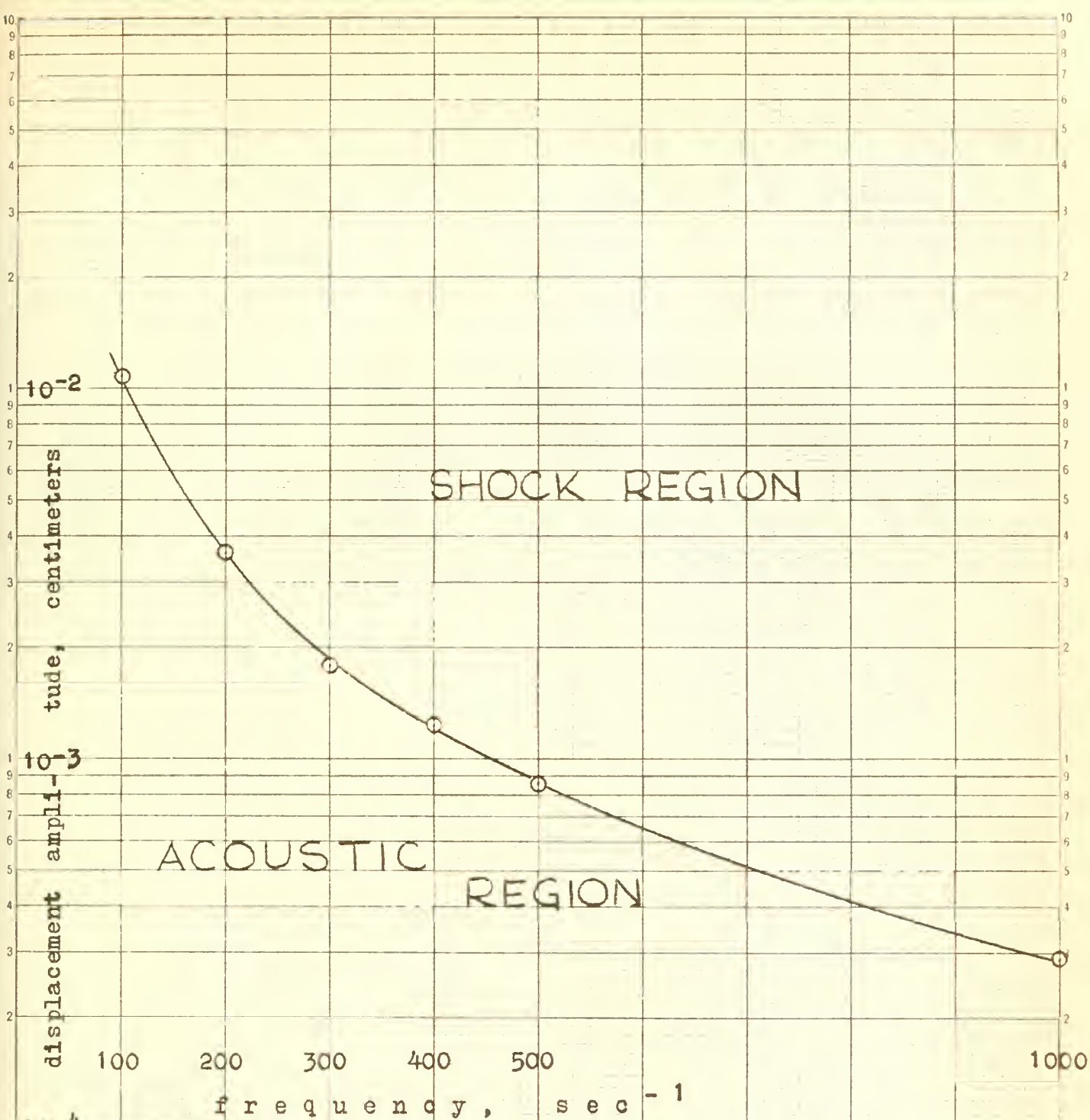RMS Acceleration for Production of
Periodic Shocks

50

Figure 25. Phase Diagram Showing Threshold Piston
Displacements for Production of Periodic
Shock Waves

6. Conclusions

The existence of acceleration as a similarity parameter has been definitely demonstrated. This fact is important when considering the use of an electrodynamic shaker as a driver for generating shock waves. Such shakers are rated at a certain maximum force output, which implies a maximum acceleration. Since acceleration is a similarity parameter, this implies that if a shaker is satisfactory for generating shock waves at one frequency, it can be used for this purpose at other frequencies within its available range. Because of the frequency limitation of the Altec microphone, the upper limit at which shocks can be generated was not found.

An examination of the plots of harmonic analysis, $\triangle$, and normalized shock strength with respect to $\mathcal{A}$ shows a tendency for these curves to level off at the higher accelerations. This fact can be used to indicate the limits of the threshold region for periodic shock waves. Unless unforseen excursions of these curves take place at higher values of $\mathcal{A}$, it can be concluded that at accelerations of approximately twice that available with the equipment used in this experiment, the curves of Figs. 6, 7, 8, 11, 12, 13, 18, 19 and 20 will have flattened out. Since the Calidyne Shaker Model A88 has available accelerations of the order of 20g, this means that a machine with rated accelerations of the order of 40g - 50g will be of sufficient capacity for the study of threshold phenomena.

7. Suggestions for Further Experiment

Saenger and Hudson [8] state that a small amount of experimental data shows qualitative agreement with their theory. Because the behavior of P_O was not measured in the present experiment, no additional evaluation of the correctness of the theory can be made here. A further step in the present process would be to obtain the variation of P_O with $\mathcal{A}$ and Δ, along with shock strength information, and use this information to obtain a quantitative evaluation of their theory.

In this process it would be necessary to determine experimental values for the constants B and Φ. It is known that the experimental and theoretical values for B are in fairly close agreement, and that this is not true for Φ. One possibility would be to use the digital computer to compare experimental data with that calculated using various values of Φ, until a Φ of optimum size to suit experimental fact could be found. Once this optimum Φ is found, it could be determined whether the theory adequately describes periodic shock waves.

The relatively slow rise time of the microphone employed in the present experiment limited the extent to which frequency variation could be investigated. The microphone rise time became a significant fraction of the period when frequency was increased to 1000 cycles per second, and no investigations above that frequency were considered worthwhile. It is recommended that an investigation be made of the effects encountered at higher frequencies, using a more suitable means of pressure measurement. If a faster-acting microphone is used to measure

pressure, it will be necessary to use a suitable amplifier, with a filter to remove the mechanical ringing inherent in such equipment.

Schlieren-optical techniques offer another means of investigating shock phenomena [10], [11]. Precise measurements of density variations in the air column can be made, and, provided that one additional parameter, such as particle velocity, is measured, the corresponding pressures can be obtained using the equation of state.

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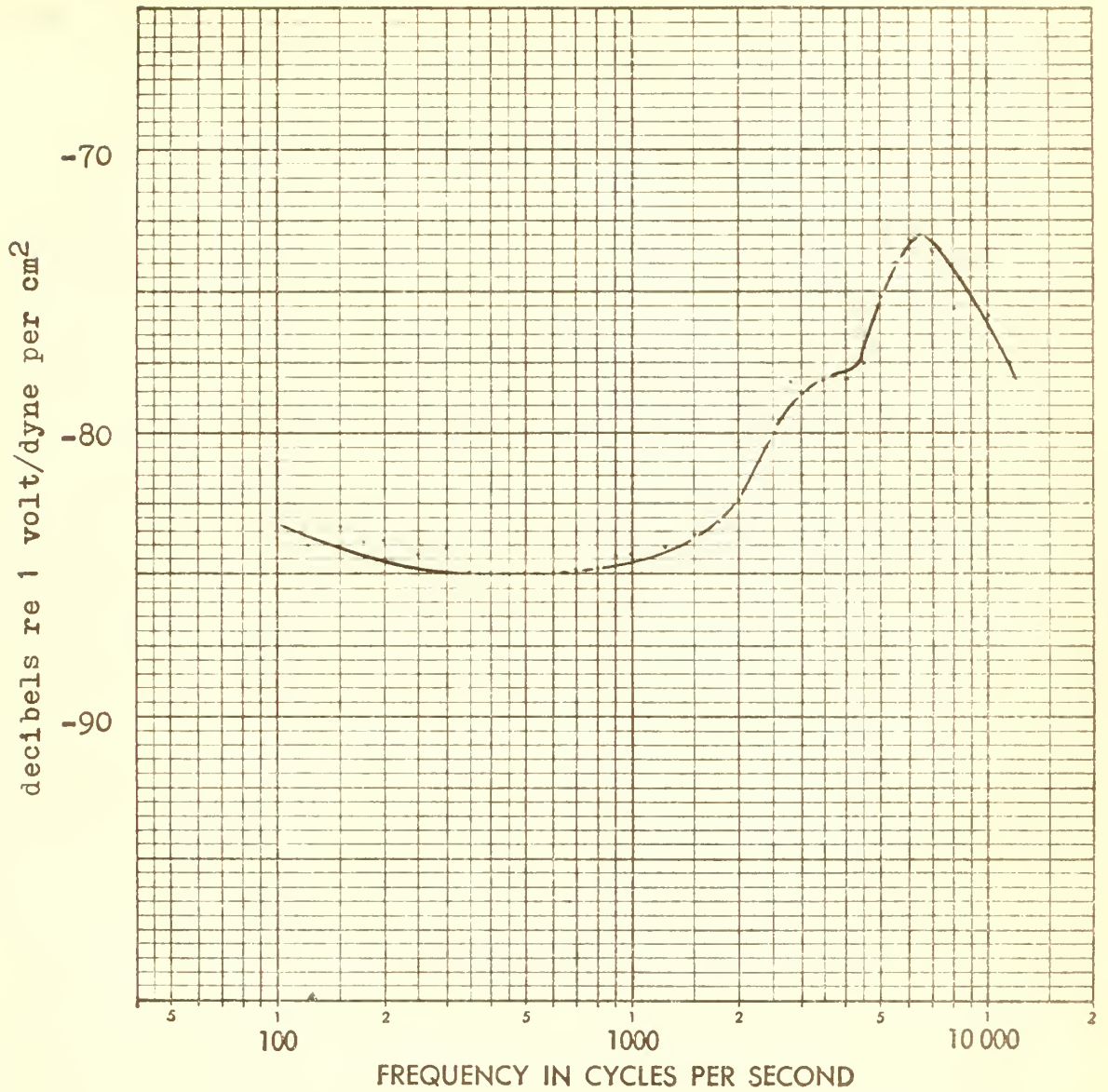
APPENDIX I

SPECIFICATIONS OF MODEL A88 SHAKER

Characteristic

Rated Force Output	100 lbs @ 5.26 amp rms
Field Current Required	5.0 amperes
Continuous Armature Current	5.3 amperes
Force/Current	19 lbs/ampere rms
Longitudinal Resonance	2560 cps
Armature Weight	2.68 lbs
Stroke	1 inch peak-to-peak
Velocity Signal Calibration	0.110 volt/inch per sec.

Microphone Calibration Curve

APPENDIX III

TABLE OF SYMBOLS

$\mathcal{A}$	Root mean square piston acceleration
a	Radius of tube
B	Dimensionless form of proportionality constant in Newton's law of cooling
c	Velocity of sound
f	Driving frequency of piston
f_1	Fundamental resonance frequency of tube
L	Tube length measured from mean position of piston to closed end of tube
ΔP	Shock strength -- the positive jump in pressure across the shock.
P_0	Instantaneous pressure at a point in the tube where gas is at rest
u	Gas particle velocity
V	Piston velocity amplitude
x	Coordinate of position along length of tube
X	Piston displacement amplitude
γ	Ratio of specific heats, c_p/c_v
Δ	Phase lag of the piston in its outstroke (maximum withdrawn) position, behind the appearance of the shock at the piston.
Δ_{max}	The value of Δ measured with the shaker operating at its maximum available $\mathcal{A}$.
λ	Wavelength
μ	Viscosity of gas
ν	Kinematic viscosity
ρ	Density of gas
Φ	Dimensionless form of viscosity parameter
ψ'	$-\left[(\gamma-1)B^2 - 2(\gamma-1)B\Phi - \Phi^2\right] / \left[(\gamma-1)B + \Phi\right]$